UNIT - IV

CONTINUOUS - TIME LTI SYSTEM:

Continuous time system & its properties. Linear time Invariant (LTI) system, Impulse response. convolution. Analysis of LTI system using Laplace & Fourier transforms.

INTRODUCTION :-

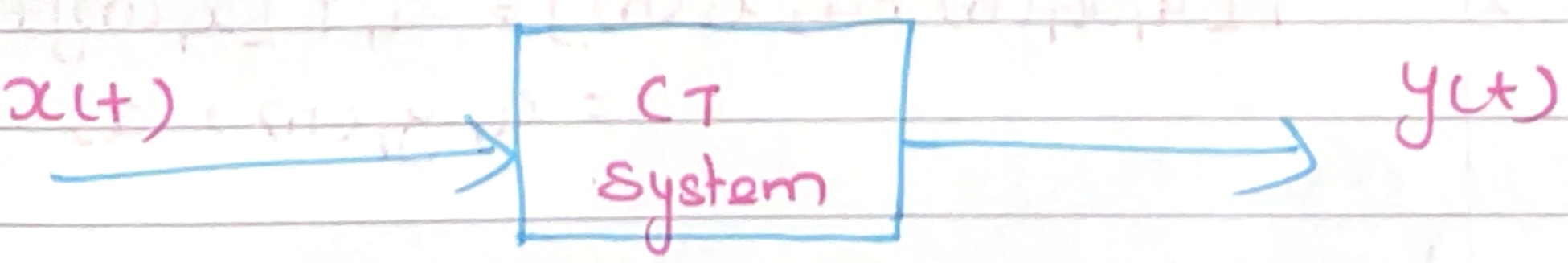
* As discussed earlier systems are broadly classified as continous time systems & discrete time system.

* Continuous time systems deal with continuous time signals & Discrete time systems deal with discrete time signals.

* Both CT & DT systems have several basic properties. out of these several basic properties of system, two properties namely linearity & time Invariance play vital role in the analysis of signals & systems.

Continuous time system & its properties :-

The input & output of a system are both continous time (CT) signals.



$$y(t) = T[x(t)]$$

eg: i) Audio - video amplifier
ii) Power supply unit
iii) RLC circuit.

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Page No.:

Date:

YOUVA

Properties of system:

* Linear & Non linear system
* Time variant & Invariant system
* Static & Dynamic system
* Causal & Non causal system
* Stable & unstable system.

LINEAR SYSTEMS AND NON LINEAR SYSTEMS:

A system is linear if it satisfies the linearity property (or) the super position principles ie):

* A linear system follows the law of superposition.
* This law is necessary & sufficient to prove linearity of system.
* Its a combination of two laws:
  (i) law of additivity
  (ii) law of homogenity.

Mathematically,

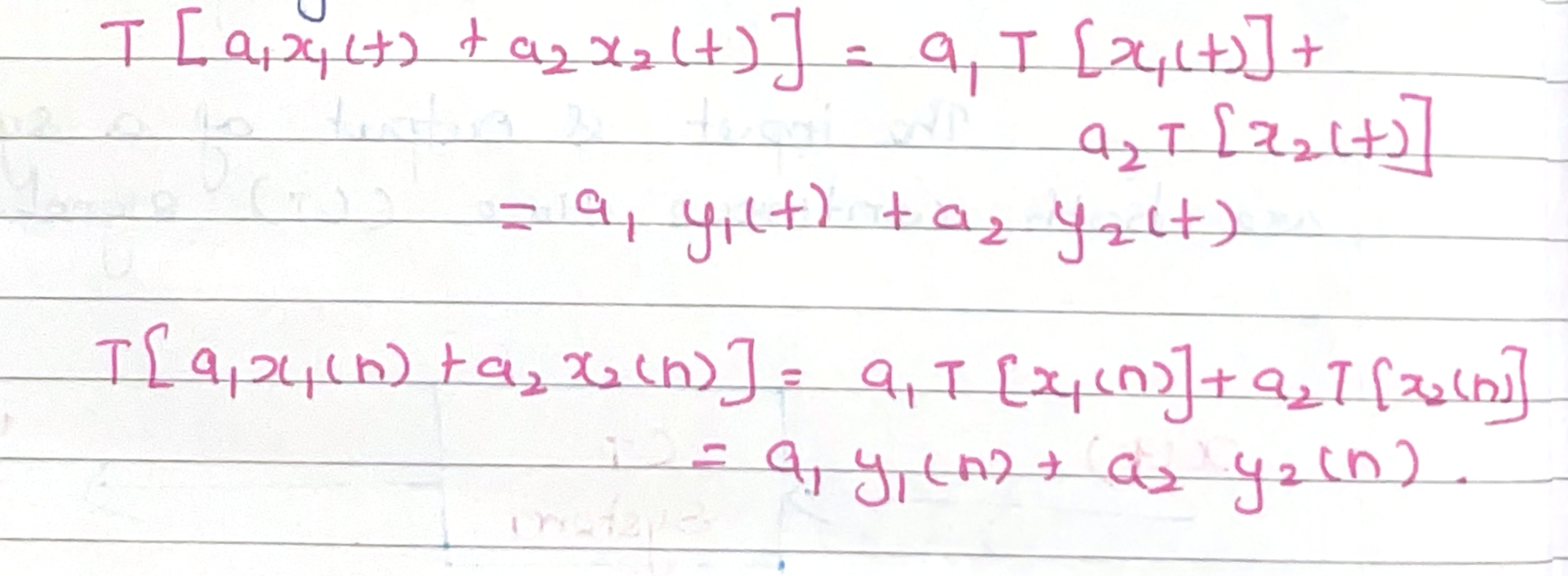
for CT → $T[a_1 x_1(t) + a_2 x_2(t)] = a_1 T[x_1(t)] + a_2 T[x_2(t)]$

$= a_1 y_1(t) + a_2 y_2(t)$

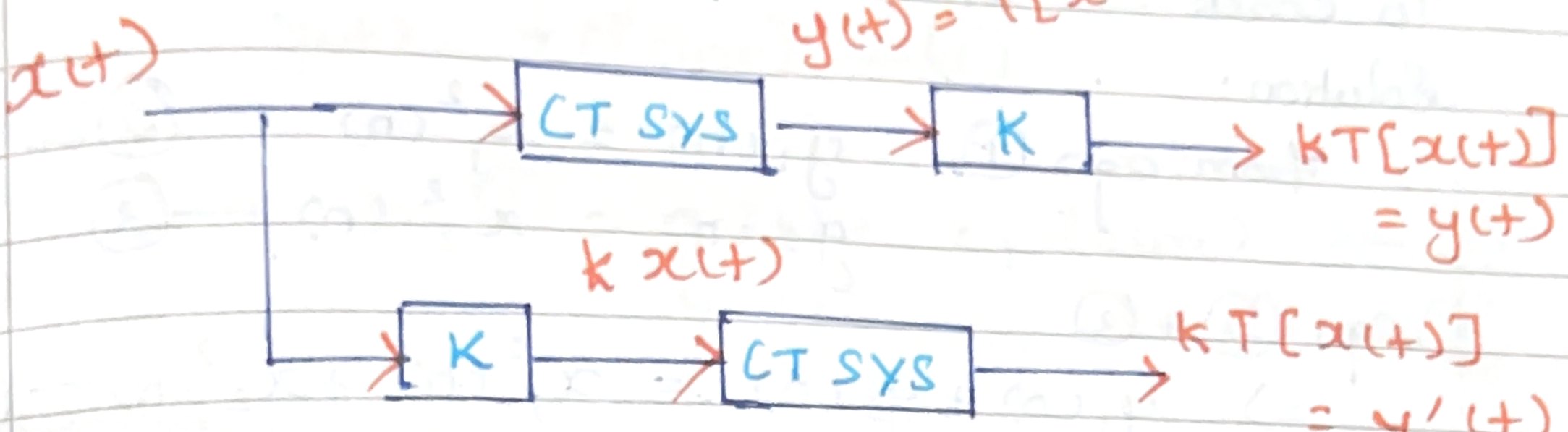
for DT → $T[a_1 x_1(n) + a_2 x_2(n)] = a_1 T[x_1(n)] + a_2 T[x_2(n)]$

$= a_1 y_1(n) + a_2 y_2(n)$.

Law of ADDITIVITY:



LAW OF HOMOGENITY:



Problems :-

1) Check whether the following systems are linear (or) not.

a) $y(t) = t\,x(t)$ b) $y(n) = x^2(n)$

c) $y(t) = x(t) + 10$ d) $y(t) = x(\sin t)$

e) $y(n) = \sin(x(n))$ f) $y(t) = e^{x(t)}$

Solution:

a) Given:

$$y(t) = t\,x(t) \quad -(1)$$

To find :

linear (or) not

Sol^n: From eqn (1) $y_1(t) = t\,x_1(t) \quad -(2)$

$$y_2(t) = t\,x_2(t) \quad -(3)$$

(2) + (3)

$$y_1(t) + y_2(t) = t\,x_1(t) + t\,x_2(t)$$
$$= t[x_1(t) + x_2(t)] \quad -(4)$$

Replace $x(t) = x_1(t) + x_2(t)$ in eqn (1)

$$(1) \Rightarrow y(t) = t[x_1(t) + x_2(t)] \quad -(5)$$

from eqn (5) & (4)

$$y(t) = y_1(t) + y_2(t)$$

$\therefore$ The given system $y(t) = t\,x(t)$ is a linear system.

b) Given:

$$y(n) = x^2(n) \quad -(1)$$

To check: linear (or) not

Solution:

From eqn (1) $y_1(n) = x_1^2(n)$ — (2)

$y_2(n) = x_2^2(n)$ — (3)

eqn (2) + (3)

$\Rightarrow y_1(n) + y_2(n) = x_1^2(n) + x_2^2(n)$ — (4)

Replace $x(n)$ by $[x_1(n) + x_2(n)]$ in eqn (1)

(1) $\Rightarrow y(n) = [x_1(n) + x_2(n)]^2$

$= x_1^2(n) + x_2^2(n) + 2x_1(n)x_2(n)$ — (5)

From eqn (5) & (4)

$y(n) \neq y_1(n) + y_2(n)$.

$\therefore$ The given system $y(n) = x^2(n)$ is a non-linear system.

1(c) Given:

$y(t) = x(t) + 10$ — (1)

To find: linear (or) not

Soln:

From eqn (1) $y_1(t) = x_1(t) + 10$ — (2)

$y_2(t) = x_2(t) + 10$ — (3)

eqn (2) + (3)

$\Rightarrow y_1(t) + y_2(t) = x_1(t) + 10 + x_2(t) + 10$

$= x_1(t) + x_2(t) + 20$ — (4)

Replace $x(t) = x_1(t) + x_2(t)$ in eqn (1)

(1) $\Rightarrow y(t) = x_1(t) + x_2(t) + 10$ — (5)

From eqn (4) & (5)

$y(t) \neq y_1(t) + y_2(t)$

$\therefore$ The given system $y(t) = x(t) + 10$ is non-linear system.

d) Given:

$$y(t) = x([\sin t]) \quad \text{— (1)}$$

Soln:

from eqn (1) $y_1(t) = x_1(\sin t)$ — (2)

$y_2(t) = x_2(\sin t)$ — (3)

(2) + (3)

$\Rightarrow y_1(t) + y_2(t) = x_1(\sin t) + x_2(\sin t)$ — (4)

Replace $x(t) = x_1(t) + x_2(t)$ in eqn (1)

(1) $\Rightarrow y(t) = x_1(\sin t) + x_2(\sin t)$ — (5)

eqn (4) & (5) from.

$$y(t) = y_1(t) + y_2(t)$$

∴ The given signal $y(t) = x(\sin t)$ is a linear system.

e) Given:

$$y(n) = \sin[x(n)] \quad \text{— (1)}$$

Soln:

From eqn (1) $y_1(n) = \sin(x_1(n)]$ — (2)

$y_2(n) = \sin(x_2(n)]$ — (3)

eqn (2) + (3)

$\Rightarrow y_1(n) + y_2(n) = \sin(x_1(n)) + \sin(x_2(n)]$ — (4)

Replace $x(n) = x_1(n) + x_2(n)$ in eqn (1)

(1) $\Rightarrow y(n) = \sin(x_1(n) + x_2(n)]$ — (5)

from (4) & (5)

$y(n) \neq y_1(n) + y_2(n)$

∴ The given signal $y(n) = \sin[x(n)]$ is a NON. linear system.

8) Given:

$$y(t) = e^{x(t)} \quad \text{— (1)}$$

soln:

from eqn (1) $y_1(t) = e^{x_1(t)}$ — (2)

$y_2(t) = e^{x_2(t)}$ — (3)

(2) + (3)

$$\Rightarrow y_1(t) + y_2(t) = e^{x_1(t)} + e^{x_2(t)} \quad \text{— (4)}$$

Replace $x(t) = x_1(t) + x_2(t)$ in eqn (1)

$$(1) \Rightarrow y(t) = e^{x_1(t) + x_2(t)} \quad \text{— (5)}$$

From (4) & (5)

$$y(t) \neq y_1(t) + y_2(t)$$

∴ The given system $y(t) = e^{x(t)}$ is a NON-linear system.

TIME INVARIANT AND TIME VARIANT SYSTEM:-

* A system is time variant if its behaviours & Input & output characteristics varying / changing with respect to time.

Mathematically,

If $y(t) = T[x(t)]$

for CT → then $T[x(t-T_0)] \neq y(t-T_0)$

for DT → If $y(n) = T[x(n)]$

then $T[x(n-N_0)] \neq y(n-N_0)$

eg: ✓ RC circuits

✓ RLC circuits.

* A system is time INvariant

if its behaviour & input-output characteristics does not vary / change w.r.to time.

Mathematically,

for CT → If $y(t) \neq T[x(t)]$
then $T[x(t-T_0)] = y(t-T_0)$

for DT → If $y(n) \neq T[x(n)]$
then $T[x(n-N_0)] = y(n-N_0)$

eg: ✓ Differentiator

Problems:
Check whether the following systems are time-variant (or) not.
a) $y(t) = \sin x(t)$ b) $y(n) = n x(n)$

a) Given
$y(t) = \sin x(t)$ — (1)
To find: Time variant (or) not
Soln:
Replace t by $(t-T_0)$ in eqn (1) on i/p alone.

(1) $\Rightarrow y(t) = \sin[x(t-T_0)]$ — (2)
Replace t by $(t-T_0)$ in eqn (1)

(1) $\Rightarrow y(t-T_0) = \sin[x(t-T_0)]$ — (3)

From eqn (2) & (3)

$$y(t) = y(t-T_0).$$

∴ The given system $y(t) = \sin x(t)$ is a time invariant.

b) Given:

$$y(n) = n\,x(n) \quad \text{— (1)}$$

To check: Time variant (or) Not

Soln:

Replace 'n' by $(n - N_0)$ in eqn (1) on i/p alone

(1) $\Rightarrow$ $y(n) = n\,x(n-N_0)$ — (2)

Replace n by $(n-N_0)$ in eqn (2)

$$y(n-N_0) = n-N_0\; x(n-N_0) \quad \text{— (3)}$$

From (2) & (3)

$$y(n) \neq y(n-N_0)$$

$\therefore$ The given system $y(n) = n\,x(n)$ is Time Variant.

Causal and Non Causal system :-

✓ * If the ouput of the system depends only on present & past values of input but not depends on future value of i/p then the system is called Causal system.

* causal system are physically realisable.

eg: Resistor, Delay unit.

✓ * If the output of the system depends only on future values of input then the system is called Non-causal system.

* It cannot by implemented practically. res; It can be produce its Output before input is applied.

eg: Image processing System.

Problems:

Check whether the following systems are causal (or) not.

(i) $y(t) = x(t)$ (ii) $y(t) = x(t-1)$

(iii) $y(n) = x(n) + x(n-1)$ (iv) $y(t) = x(t+1)$

(v) $y(n) = x(n-1)$.

(i) Given:

$$y(t) = x(t) \quad \text{— (1)}$$

To check: causal (or) not

Soln:

Put $t = -1, 0, 1, 2$ in eqn (1)

(1) =>

$$y(-1) = x(-1)$$
$$y(0) = x(0)$$
$$y(1) = x(1)$$
$$y(2) = x(2)$$

For all time, output depends on present i/p.

∴ The given system $y(t) = x(t)$ is a causal system.

(ii) Given:

$$y(t) = x(t-1) \quad \text{— (1)}$$

To check: causal (or) not

Soln:

put => $t = -1, 0, 1, 2$ in eqn (1)

(1) =>

$y(-1) = x(-2)$ (present, past)

$y(0) = x(-1)$ (present, past)

$y(1) = x(0)$ (present, past)

$y(2) = x(1)$ (present, past)

For all time, o/p of a system depends on past values of i/p

∴ The given system $y(t) = x(t-1)$ is a causal system.

Static & Dynamic system:

* If the o/p of the system depends only on present values of i/p at each & every instant of time then the system is called static.

* also known as memory less system - these systems contains no energy storage elements

eg: Resistive system.

* If the output of the system depends on past (or) future values of i/p at any instant of time then the system is called Dynamic.

* also known as Memory system. these system contain one (or) more storage elements.

eg: Capacitive system.

Problem:
Check whether the following systems are static (or) Dynamic.

(i) $y(t) = x(t) + x(t-1)$

(ii) $y(t) = x(\sin t)$

(iii) $y(n) = \text{Even}[x(n)]$.

Stable & unstable system:

* If a system satisfies the BIBO (Bounded Input for bounded Output) stability condition then it is calbd stable system.

* BIBO stability condition is given by,

for CT $$\int_{-\infty}^{\infty} |h(t)|\,dt < \infty$$

for DT $$\sum_{n=-\infty}^{\infty} |h(n)|\,dt < \infty$$

where $h(t)$ / $h(n)$ → Impulse response of the System.

* If the system does not satisfy the BIBO condition then it is called unstable system

Problems:

Check whether the following system are stable (or) not:

(i) $y(t) = x(t) + 2$ (ii) $y(n) = n\,x(n)$.

1) Given:

$$y(t) = x(t) + 2$$

To check: stable (or) Not

Soln:

$$y(t) = x(t) + 2$$

If $x(t) = 10 \Rightarrow y(t) = 10 + 2 = 12$

∴ The given system $y(t) = x(t) + 2$ is a stable system.

ii) Given $y(n) = nx(n)$

To check: stable (or) NOT.

Page No.:
Date:
YOUVA

Soln:

$$y(n) = n\,x(n)$$

If $x(n) = 5$

$$\therefore y(n) = 5n \Big\langle_{-\infty}^{\infty}$$

$y(n)$ is not a finite.

∴ The given system $y(n) = n\,x(n)$ is a unstable system.

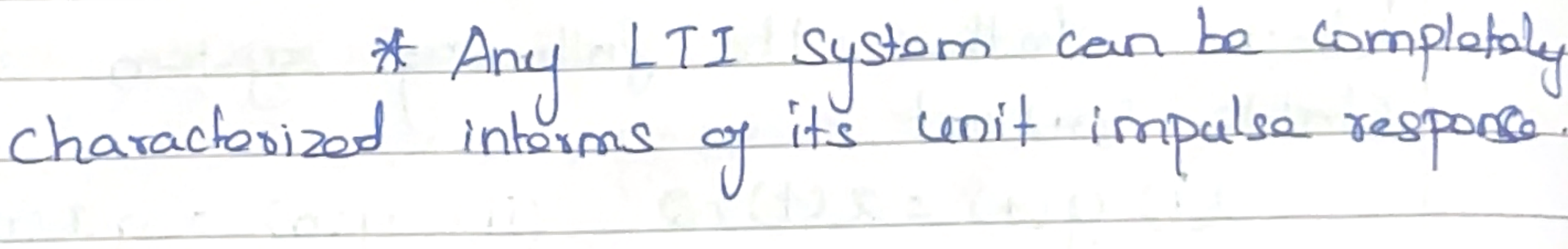
Linear Time Invariant (LTI) System:

* If a system has both the linearity (satisfies law of additivity and law of homogeneity) and time invariance (output of a system does not vary with respect to time) properties, then this system is called linear time invariant (LTI) system.

* Any LTI system can be completely characterized interms of its unit impulse response.

IMPULSE RESPONSE:

* The output response of LTI system to an unit impulse is called as impulse response (or) unit impulse response.



i.e) The impulse response $h(t)$ of a fixed linear system is the response of the system to unit impulse applied at $t = 0$ with zero initial conditions.

CONVOLUTION INTEGRAL :

* The convolution between input signal $x(t)$ & impulse response $h(t)$ of LTI-CT system is called the convolution integral & defined mathamatically by,

$$y(t) = x(t) * h(t) = \int_{\tau=-\infty}^{\infty} x(\tau) h(t-\tau) d\tau$$

Properties of convolution integral :

* 1> Commutative property :

$$y(t) = x(t) * h(t) = h(t) * x(t)$$

* 2> Distributive property :

$$y(t) = x(t) * [h_1(t) + h_2(t)] = [x(t) * h_1(t)] + [x(t) * h_2(t)]$$

* 3> Associative property :

$$y(t) = x(t) * [h_1(t) * h_2(t)] = [x(t) * h_1(t)] * h_2(t).$$

NOTE:

* A continuous time LTI system is known as causal if its impulse response, $h(t)$ is zero for $t<0$

* A continuous time LTI system is known as stable if its impulse response satisfies the condition.

$$\int_{-\infty}^{\infty} h(t)\, dt < \infty$$

Page No.:
Date:
YOUVA

Problems:

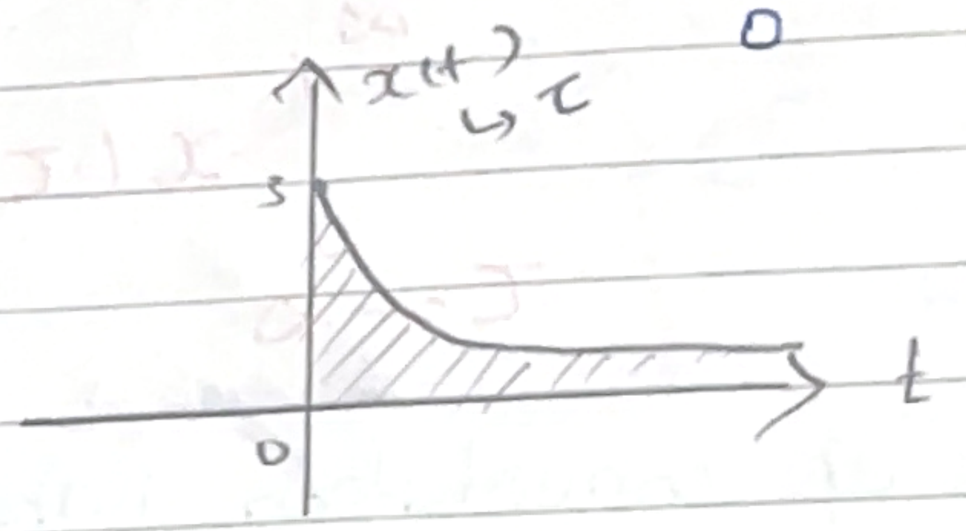
(1) obtain the convolution of the two continuous time signals given below:

$x(t) = e^{-3t} u(t)$ & $h(t) = u(t-1)$

Given:

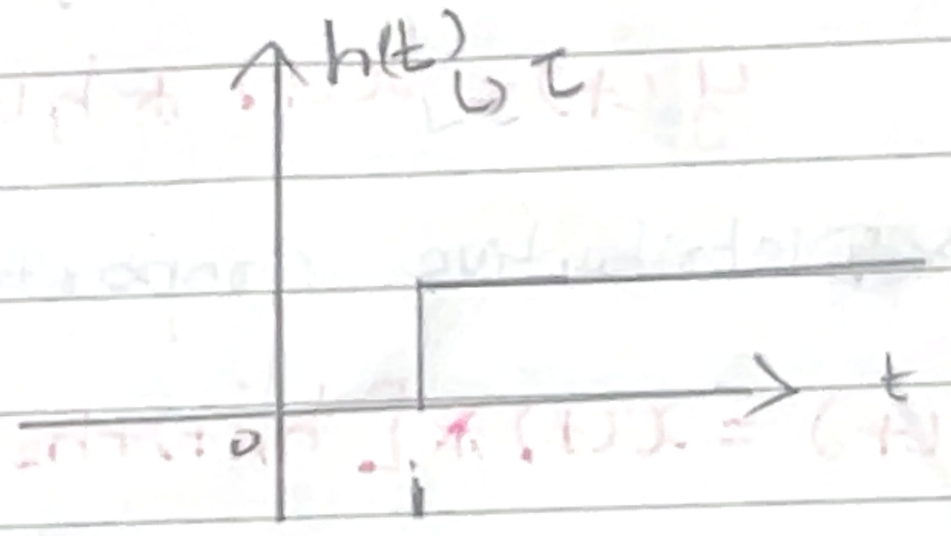
$x(t) = e^{-3t} \quad u(t) = e^{-3t} \; ; t \geq 0$

$0 \; ; t < 0$



$h(t) = u(t-1) = 1 \; ; t \geq 1$

$0 \; ; t < 0$

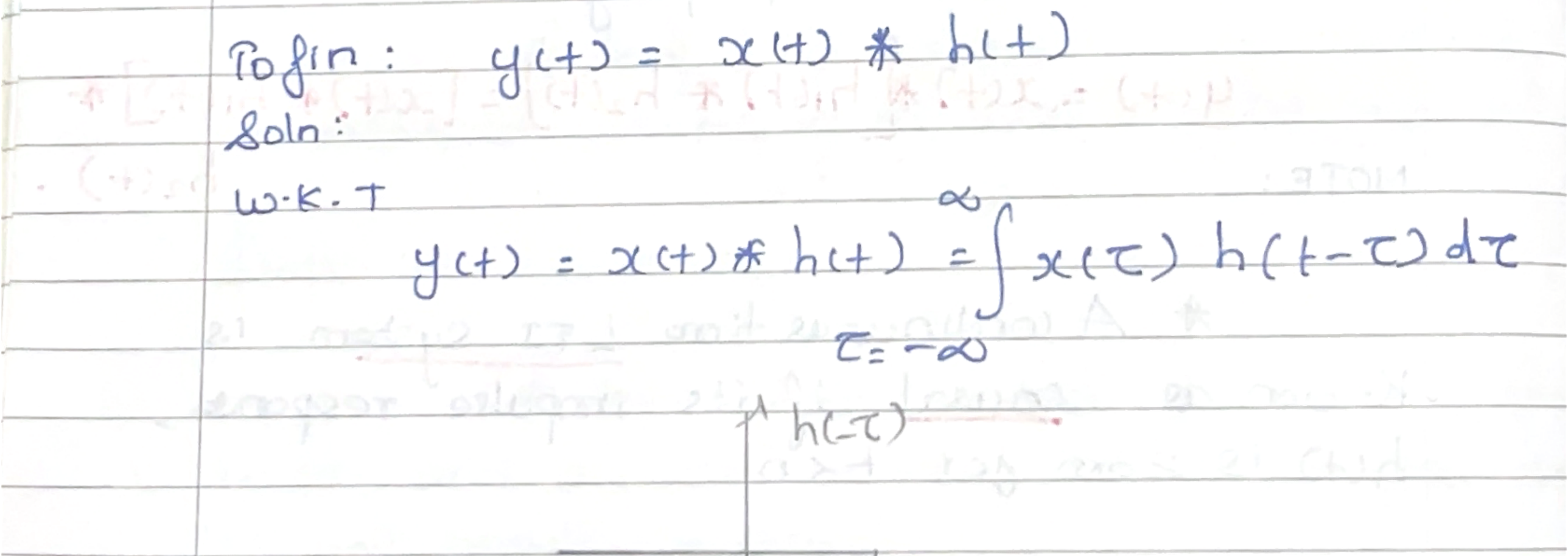


To find: $y(t) = x(t) * h(t)$

Soln:

W.K.T

$$y(t) = x(t) * h(t) = \int_{\tau=-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$



$$= \int_{\tau=1-t=0}^{t-1} e^{-3\tau} \times 1 \; d\tau$$

$$= \left[\frac{e^{-3\tau}}{-3} \right]_0^{t-1}$$

$$= -1/3 \left[e^{-3(t-1)} - e^{0} \right]$$

$$y(t) = -1/3 \left[1 - e^{-3(t-1)} \right]$$

(2) By using Continuous-time convolution integral, find the response of the system to unit step input signal. Impulse response is given as:

$$h(t) = R/L \; e^{-R/L\, t} \; u(t)$$

Given:

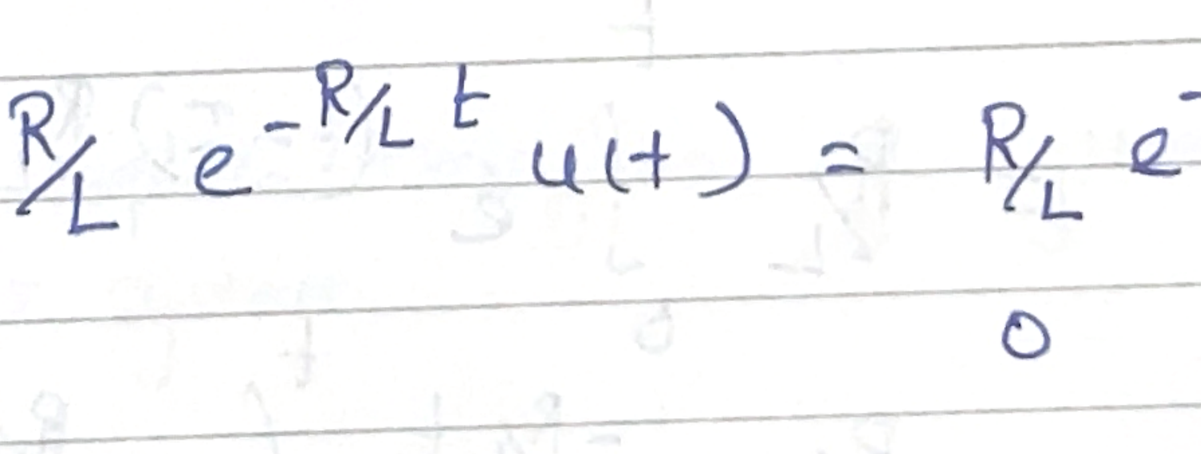
$$x(t) = u(t) = 1 \; ; \; t \geq 0$$
$$0 \; ; \; t < 0$$

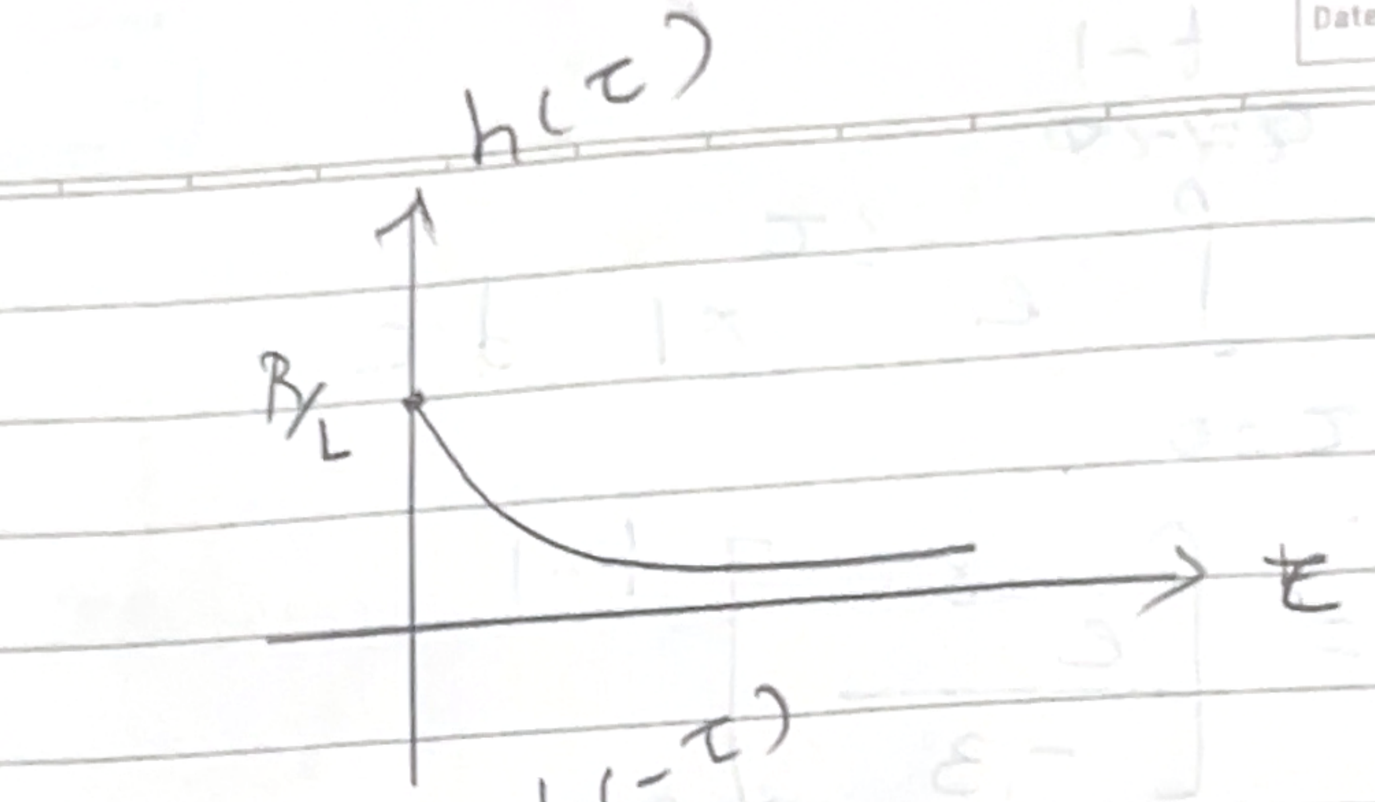
$$h(t) = R/L \; e^{-R/L\, t} \; u(t) = R/L \; e^{-R/L\, t} \; ; \; t \geq 0$$
$$0 \; ; \; t < 0$$

Soln:

W.K.T

$$y(t) = x(t) * h(t) = \int_{\tau=-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$





$$= \int_{\tau=0}^{t} 1 \times R/L \; e^{-(t-\tau) R/L} \, d\tau$$

$$= R/L \int_{0}^{t} e^{-(t-\tau) R/L} \, d\tau$$

$$= R/L \; e^{-R/L\, t} \int_{0}^{t} e^{R/L\, \tau} \, d\tau$$

$$= R/L \; e^{-R/L\, t} \left[\frac{e^{R/L\, \tau}}{R/L} \right]_0^t$$

$$= \cancel{R/L} \; e^{-R/L\, t} \; \cancel{L/R} \left[e^{R/L\, t} - e^{0} \right]$$

$$y(t) = 1 - e^{-R/L\, t} \quad , t \geq 0$$

Analysis of LTI - CT system using laplace & fourier transform:

Transfer function:

✓ Also known as system function and defined as the ratio between the transform of output, $y(t)$ & input $x(t)$ of a system.

✓ It is represented by 'H' and using Laplace transform, the transfer function is defined by.

$$H(s) = \frac{L[y(t)]}{L[x(t)]} = \frac{Y(s)}{X(s)}$$

Impulse Response:

↑ The impulse response of LTI - CT system can be represented as $h(t)$ and defined as,

$$h(t) = L^{-1}[H(s)] = L^{-1}\left[\frac{Y(s)}{X(s)}\right]$$

ie; Inverse Laplace transform of system function of a LTI - CT system.

(OR)

The output response of LTI-CT system when input as a unit impulse signal is known as Impulse Response is;

W.K.T

$$Y(s) = H(s)X(s)$$

$$\Rightarrow y(t) = L^{-1}[H(s)X(s)]$$

$$y(t) = h(t) = L^{-1}[H(s)X(s)]$$

where

$$X(s) = L[x(t)] = L[\delta(t)]$$

Note:

The output, $y(t)$ of LTI - CT System when input, $x(t)$ as a unit step signal, $u(t)$ is known as Step Response ie;

$$y(t) = L^{-1}[H(s)X(s)] \; ; \; X(s) = L[u(t)]$$

Properties of Impulse Response:

* For a LTI - CT system to be causal,

$$h(t) = 0 \; ; \; t < 0$$

ie; An impulse response of a causal system is also called causal.

* A LTI System is Stable if,

$$\int_{-\infty}^{\infty} |h(t)| \, dt < \infty$$

ie; The BIBO (Bounded input for bounded output) stable system has absolutely integrable impulse response.

Problems:-

(1) The system produces the output $y(t) = e^{-t}u(t)$ for an input $x(t) = e^{-2t}u(t)$. Determine the impulse & frequency response of the system.

Given: $x(t) = e^{-2t}u(t)$

$y(t) = e^{-t}u(t)$

To find:

Impulse response, $h(t)$

Frequency Response; $H(j\omega)$

Soln:-

w.k.t $h(t) = L^{-1}[H(s)]$

where $H(s) = \frac{Y(s)}{X(s)} = \frac{L[y(t)]}{L[x(t)]}$

$Y(s) = L[y(t)] = L[e^{-t}u(t)]$

$Y(s) = \frac{1}{s+1}$

$X(s) = L[x(t)] = L[e^{-2t}u(t)]$

$X(s) = \frac{1}{s+2}$

$H(s) = \frac{1/s+1}{1/s+2} = \frac{1}{s+1}^{s+2} = \frac{s+2}{s+1}$ ——— (I)

$\Rightarrow h(t) = L^{-1}[H(s)] = L^{-1}\left[\frac{s+2}{s+1}\right]$ — (1)

Let $\frac{s+2}{s+1} = \frac{A}{s+1}$

$\Rightarrow s+2 = A$

put $s=0 \Rightarrow \boxed{A=2}$

$\therefore (1) \Rightarrow h(t) = L^{-1}\left[\frac{2}{s+1}\right]$

$= 2L^{-1}\left[\frac{1}{s+1}\right]$

$h(t) = 2e^{-t}u(t)$

$\because L^{-1}\left[\frac{1}{s+a}\right] = e^{-at}u(t)$

Replace $s=j\omega$ in eqn I

$\therefore (I) \Rightarrow H(j\omega) = \frac{j\omega+2}{j\omega+1}$

② Determine the transfer function and impulse response of the system having following differential equation.

$$\frac{d^2}{dt^2}y(t) + 3\frac{d}{dt}y(t) + 2y(t) = 2\frac{d}{dt}x(t) + x(t)$$

To find: Transfer function, $H(s) = ?$

Impulse response, $h(t) = ?$

Given:

$$\frac{d^2}{dt^2}y(t) + 3\frac{d}{dt}y(t) + 2y(t) = \frac{2d}{dt}x(t) + x(t) \quad \text{---- (1)}$$

Soln:

w.k.T

$$H(s) = \frac{Y(s)}{X(s)}$$

on taking LT bothsides in eqn (1)

$$L\left[\frac{d^2}{dt^2}y(t) + 3\frac{d}{dt}y(t) + 2y(t)\right] = L\left[2\frac{d}{dt}x(t) + x(t)\right]$$

ie):

$$L\left[\frac{d^2}{dt^2}y(t)\right] + 3L\left[\frac{d}{dt}y(t)\right] + 2L[y(t)] =$$

$$2L\left[\frac{d}{dt}x(t)\right] + L[x(t)]$$

$$s^2y(s) + 3sy(s) + 2y(s) = 2sx(s) + x(s)$$

$$y(s)\cdot[s^2+3y+2] = x(s)[2s+1]$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{2s+1}{s^2+3y+2} \quad \text{---- (2)}$$

w.k.T

Impulse response, $h(t) = L^{-1}[H(s)]$

$$h(t) = L^{-1}\left[\frac{2s+1}{s^2+3y+2}\right] \quad \text{---- (3)}$$

Let

$$\frac{2s+1}{s^2+3s+2} = \frac{2s+1}{(s+1)(s+2)}$$

$$\frac{2s+1}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2} = \frac{A(s+2)+B(s+1)}{(s+1)(s+2)}$$

$$2s+1 = A(s+2) + B(s+1)$$

Put $s = -1 \Rightarrow$

$$2(-1)+1 = A(-1+2)$$

$$\boxed{A = -1}$$

Put $s = -2 \Rightarrow$

$$2(-2)+1 = B(-2+1)$$

$$\boxed{B = 3}$$

eqn (3)

$$\Rightarrow h(t) = L^{-1}\left[\frac{-1}{s+1} + \frac{3}{s+2}\right]$$

$$= -L^{-1}\left[\frac{1}{s+1}\right] + 3L^{-1}\left[\frac{1}{s+2}\right]$$

$$h(t) = -e^{-t}u(t) + 3e^{-2t}u(t)$$

(3) The differential equation of the system is given as $\frac{d^2}{dt^2}y(t) + 5\frac{d}{dt}y(t) + 6y(t) = 2\frac{d}{dt}x(t) + x(t)$ with $y(0) = 1$ and $\frac{d}{dt}y(t)\Big|_{t=0} = 2$. Determine the output of the system for $x(t) = u(t)$

Given:

$$\frac{d^2}{dt^2}y(t) + 5\frac{d}{dt}y(t) + 6y(t) = 2\frac{d}{dt}x(t) + x(t) \quad \text{---(1)}$$

$$y(0) = 1 \; ; \; y'(0) = 2.$$

To find:

output, $y(t)$ when $x(t) = u(t)$.

$$L(y''(t)) = s^2Y(s) - sy(0) - y'(0)$$

$$L(y'(t)) = sY(s) - y(0)$$

Soln:

on taking LT both sides in eqn (1)

$$L\left[\frac{d^2}{dt^2}y(t) + 5\frac{d}{dt}y(t) + 6y(t)\right] = L\left[2\frac{d}{dt}x(t) + x(t)\right]$$

$$L\left[\frac{d^2}{dt^2}y(t)\right] + 5L\left[\frac{d}{dt}y(t)\right] + 6L[y(t)] =$$

$$2L\left[\frac{d}{dt}x(t)\right] + L[x(t)]$$

$$s^2 y(s) - s\,y(0) - y'(0) + 5\,[s\,y(s) - y(0)] + 6y(s) = 2\,[s\,x(s)] + x(s)$$

ie;

$$s^2 y(s) - s - 2 + 5s\,y(s) - 5 + 6y(s) = x(s)\,[2s+1]$$

$$\Rightarrow y(s)\,[s^2+5s+6] - (s+7) = x(s)\,[2s+1]$$

$$\Rightarrow y(s)\,(s^2+5s+6) - (s+7) = L\,[x(t)]\,[2s+1]$$

$$= L\,[u(t)]\,[2s+1]$$

$$= \frac{1}{s}\,[2s+1]$$

$$\Rightarrow y(s)\,(s^2+5s+6] = \frac{2s+1}{s} + (s+7)$$

$$y(s)\,[s^2+5s+6] = \frac{(2s+1)+(s+7)\,s}{s}$$

$$y(s) = \frac{2s + s + s^2 + 7s}{(s^2+5s+6)\,s}$$

$$y(s) = \frac{s^2+9s+1}{s(s^2+5s+6)}$$

$$\Rightarrow y(t) = L^{-1}[Y(s)] = L^{-1}\left[\frac{s^2+9s+1}{s(s^2+5s+6}\right] \quad (2)$$

let.

$$\frac{s^2+9s+1}{s(s+2)(s+3)} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+3}$$

ie): $s^2+9s+1 = A(s+2)(s+3) + Bs(s+3) + C(s)(s+2)$

put $s=0$ $\Rightarrow$ $1 = 6A \Rightarrow \boxed{A = 1/6}$

put $s=-2$ $\Rightarrow$ $4 - 7(-2) + 1 = B(-2)(-2+3)$

$4 - 14 + 1 = -2B$

$-9 = 2B$ $\boxed{B = 9/2}$

Put $s = -3$

$$\Rightarrow 9 - 21 + 1 = 3C$$

$$\Rightarrow -11 = 3C$$

$$\boxed{C = -11/3}$$

(2) $\Rightarrow$ $y(t) = L^{-1}\left[\frac{1/6}{s} + \frac{1/2}{s+2} + \frac{1}{s+3}\right]$

$$= 1/6\, L^{-1}\left[1/s\right] + 1/2\, L^{-1}\left[\frac{1}{s+2}\right] - \frac{11}{3} L^{-1}\left[\frac{1}{s+3}\right]$$

$$y(t) = 1/6\, u(t) + 1/2\, e^{-2t} u(t) - \frac{11}{3} e^{-3t} u(t).$$

CT system realization using laplace transform, including Direct form-I & Direct form-II

✓ A continuous time (CT) system can be represented by differential eqn. by taking the laplace transform, the diff eqn can be converted into an algebraic equation which gives the system function $H(s)$.

* Start with system equation
* convert into a form suitable for realization
* Direct form-I realization.

↗ Direct form-I Separate integrator chain for the input & feedback paths.

* Direct form-II realization.

↗ It combines the two integrator chains into one common integrator chain

Note:-

CT realization uses $1/s$ integrators

DT realization uses z^{-1} delays.

Problems :-

(1) Find the direct form-I and direct form-II structure of continuous time system represented by the equation.

$$H(s) = \frac{4s+28}{s^2+6s+5}$$

Soln:-

$$H(s) = \frac{Y(s)}{X(s)} = \frac{4s+28}{s^2+6s+5}$$

$$\frac{Y(s)}{X(s)} = \frac{s^2\left[\frac{4}{s}+\frac{28}{s^2}\right]}{s^2\left[1+\frac{6}{s}+\frac{5}{s^2}\right]}$$

$$\frac{Y(s)}{X(s)} = \frac{\frac{4}{s}+\frac{28}{s^2}}{1+\frac{6}{s}+\frac{5}{s^2}} \quad \text{---(1)}$$

Direct Form-I :

$$Y(s)\left[1+\frac{6}{s}+\frac{5}{s^2}\right] = X(s)\left[\frac{4}{s}+\frac{28}{s^2}\right]$$

$$Y(s) + \frac{6}{s}Y(s) + \frac{5}{s^2}Y(s) = \frac{4}{s}X(s) + \frac{28}{s^2}X(s)$$

$$Y(s) = \frac{4}{s}X(s) + \frac{28}{s^2}X(s) - \frac{6}{s}X(s) - \frac{5}{s^2}Y(s)$$