EC25C04

Signals and systems

UNIT I :- Introduction to signals and systems :-

Definition of signals and systems, classification of signals, operation of signals, singularity functions and related functions. Analogy between vectors and signals, orthogonal signal space, complete set of orthogonal function parseval's relations.

Definition of signals :-

Signal is a physical quantity varying with respect to one (or) more independent variables which contains some information.

ie,

A signal may be a function of time, temperature, position, pressure, distance, etc.

Eg :

* A Telephone (or) a television signal
* Music, speech, picture and video Signals.
* ECG signal - It is used to predict heart diseases.
* Electromagnetic waves.

Classification of signal :-

Based on variation in amplitude, signals are classified into mainly two types.

1. Continuous Time (CT) or Analog Signals

2. Discrete Time (DT) (or) Digital Signals.

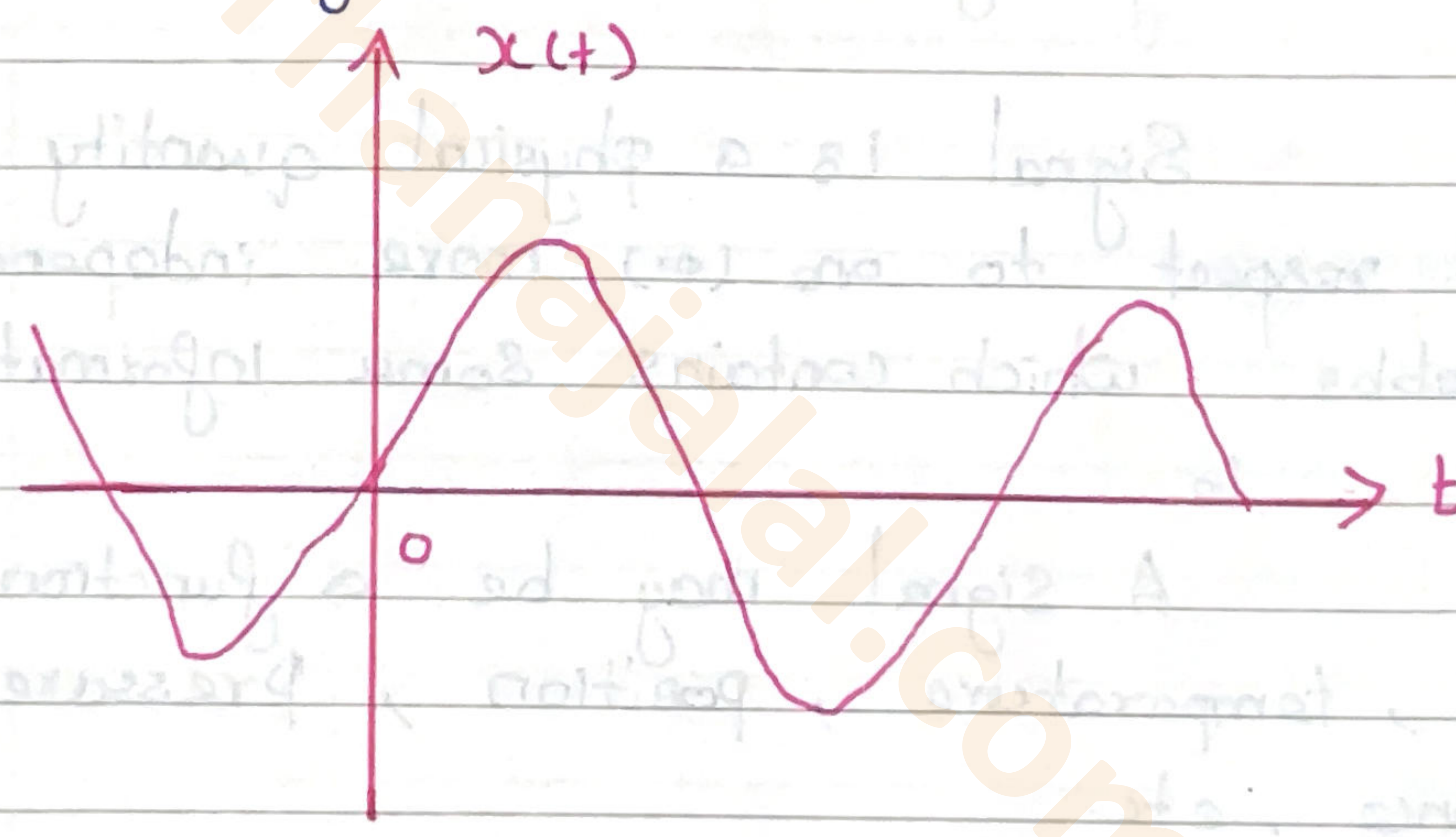
1. Continuous Time Signals :-

A signal is said to be continuous time signal if and only if it will have a strength of some value of amplitude at every instant of time

A signal of continuous amplitude and time is called as continous time signal.

eg: * A telephone (or) television signal.

* Music, speech, picture, audio & video signals.



In general, CT signals are represented as $x(t)$

2. Discrete time signals :-

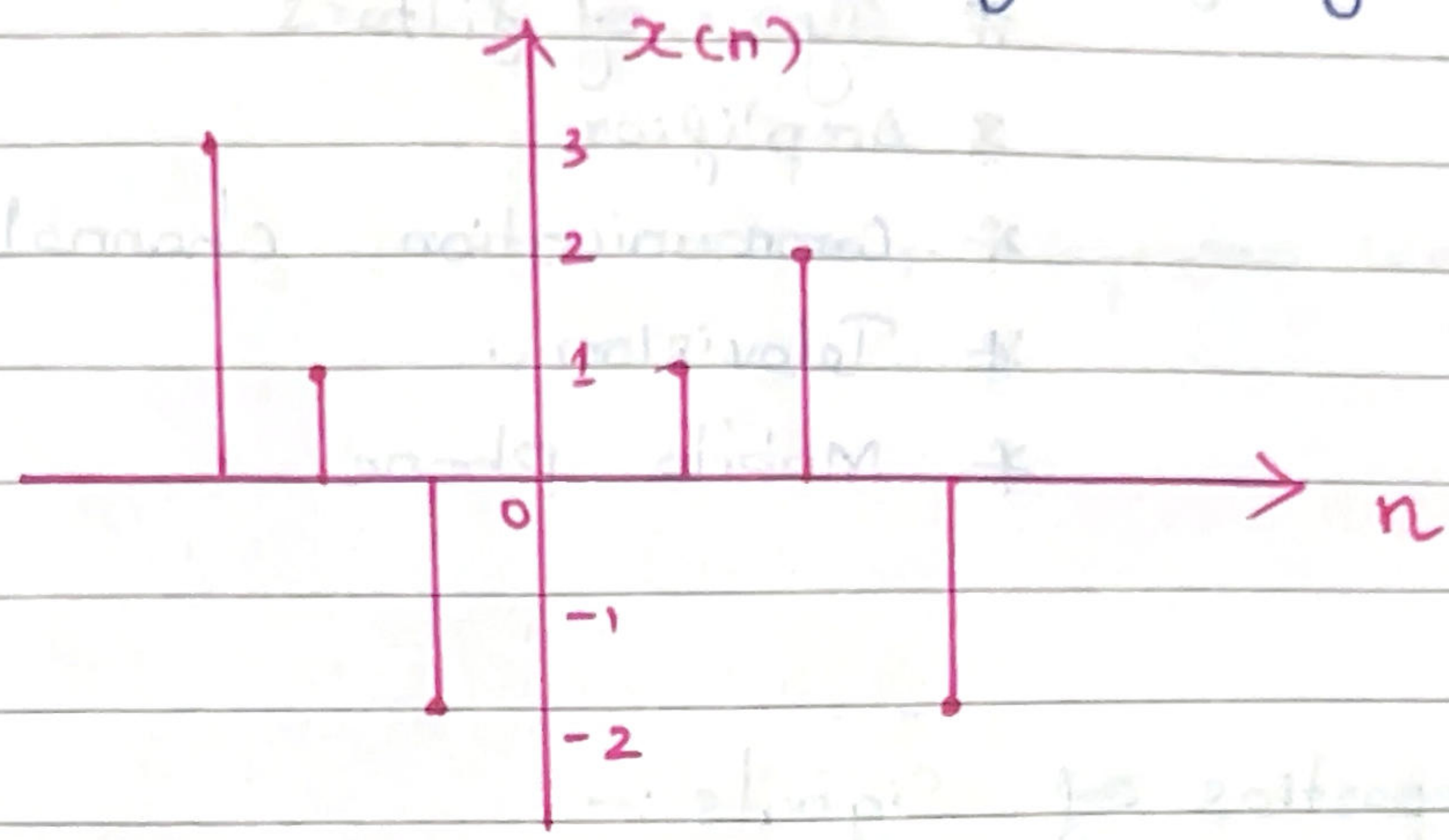
A signal is said to be discrete time (DT) signal, if and only if It will have a strength of some value of amplitude at certain instant of time.

If the signal is defined only at discrete instant of time, then it is known as Discrete time signal

eg :-

* Monthly sales of a corporation

* stock market daily averages.



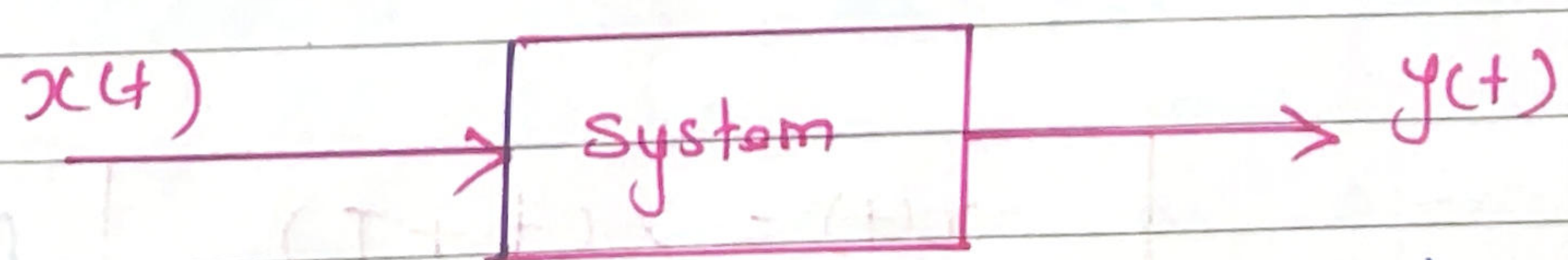
In general, DT signals are represented as $x(n)$

Definition of System :-

A system is a physical device which operates on input signal and produces an output.

A system may be defined as a set of elements (or) functional blocks which are connected together and produces an output in respect to an input signal.

The response (or) the output of the ~~signal~~ system depends upon transfer function of the system.



Mathematically, the functional relationship between input and output may be written as

$$y(t) = f\,[x(t)]$$

(or)

$$y(t) = T\,[x(t)]$$

eg:-

* Types of filters
* Amplifier
* Communication channel
* Television.
* Mobile phone..

Properties of signals :-

Based upon the property of a signal It may be classified as follows.

1. Periodic and Non periodic signals
2. Even and odd signals.
3. Energy and power signals
4. Deterministic and Random signals.
5. Causal and Noncausal signal.
6. Bounded and unbounded signal.

1. Periodic signal :-

A signal $x(t)$ is said to be periodic signal if for some positive constant T, the smallest values of T that satisfies the periodicity condition.

$$\boxed{x(t) = x(t+T)} \quad \text{for all `t'}$$

A signal which repeats ~~itself~~ itself after a fixed time period is called as a periodic signal.

T is called as fundamental period

and defined as the time taken for the signal to complete it's one cycle.

Eg:

sine, cosine, square waves.

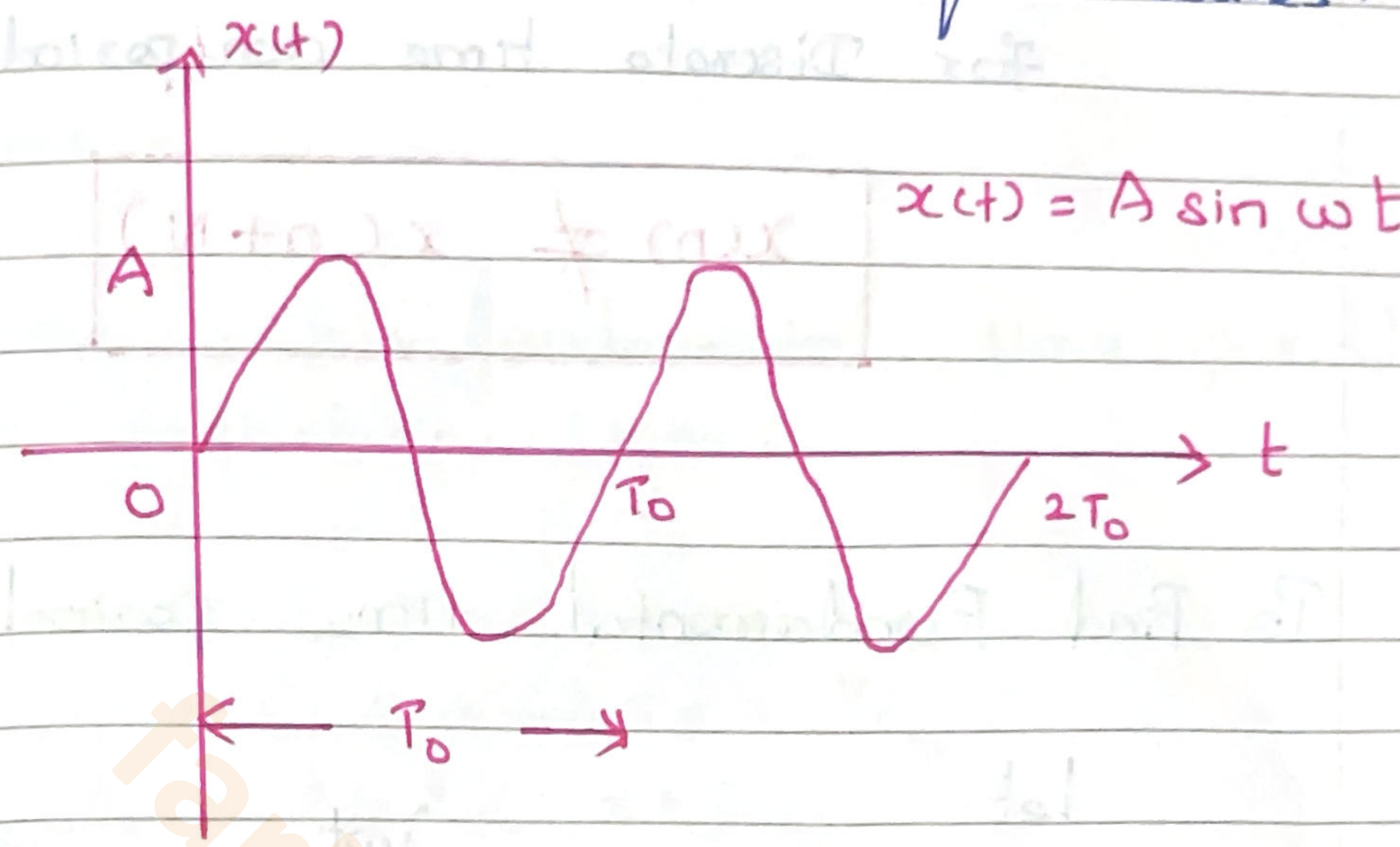


Fig: Periodic signal.

Non-periodic Signal:-

A signal $x(t)$ is said to be non-periodic signal, it does not satisfy the periodicity condition.

$$\boxed{x(t) \neq x(t+T)}$$

A signal that does not repeat itself after a fixed time period is called as non periodic signal. also known as Aperiodic signal.

eg: Exponential signal.

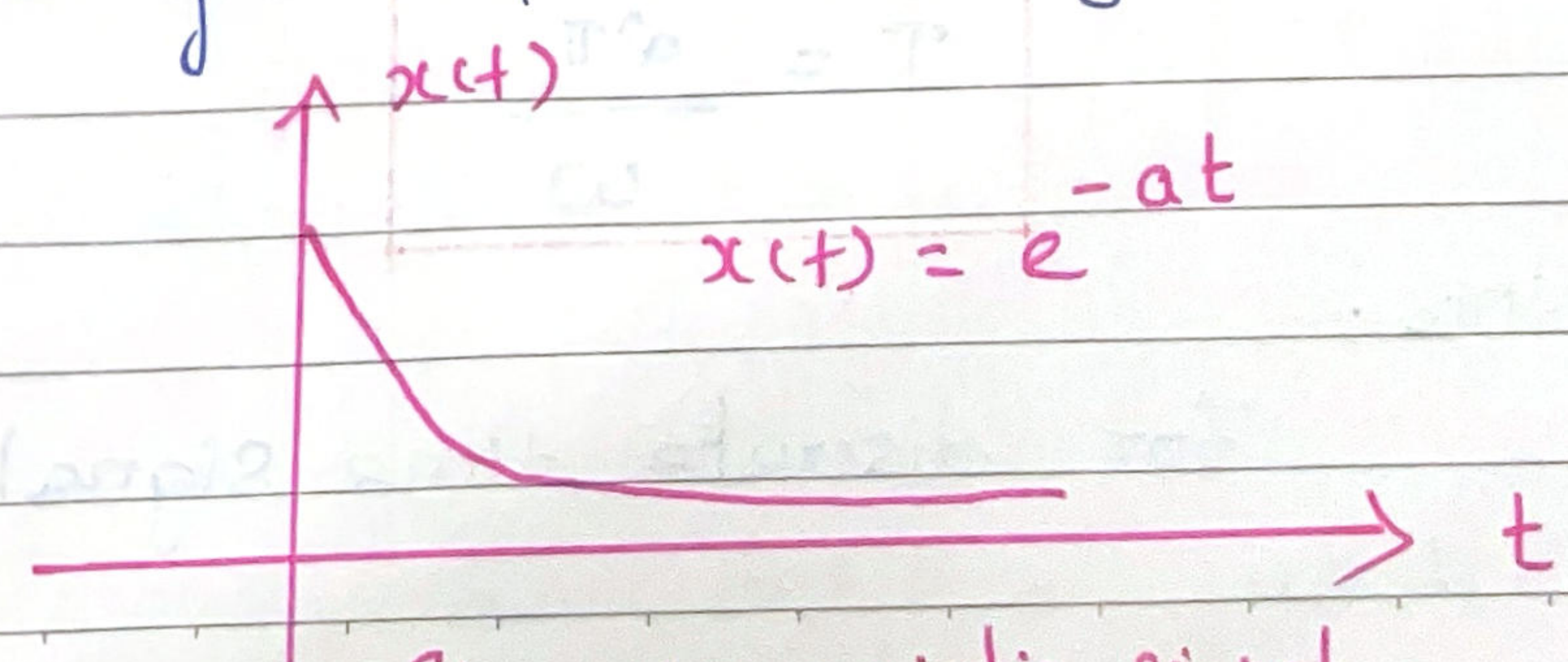


Fig) Non periodic signal.

NOTE :-

For discrete time periodic signal

$$\boxed{x(n) = x(n+N)}$$

for Discrete time non-periodic signal

$$\boxed{x(n) \neq x(n+N)}$$

To Find Fundamental time period (T)

let

$$x(t) = A e^{j\omega t} \quad \text{—— } (1)$$

$x(t)$ Satisfies the condition $x(t) = x(t+T)$

$$\therefore x(t+T) = A e^{j\omega(t+T)} \quad \text{—— } (2)$$

If periodic then (1) = (2)

$$\therefore A e^{j\omega t} = A e^{j\omega(t+T)}$$

$$\cancel{A e^{j\omega t}} = \cancel{A e^{j\omega t}} e^{j\omega T}$$

$$e^{j\omega T} = 1$$

$$e^{j\omega T} = e^{j2\pi}$$

$$\cancel{j}\omega T = \cancel{j}2\pi$$

$$\boxed{T = \frac{2\pi}{\omega}}$$

Note :-

For discrete time signal $x(n)$ the

FTP is given by

$$\boxed{N = \frac{2\pi}{\Omega}}$$

(1) Problems :-

Find the FTP (fundamental time period) of the following signals.

(i) $x_1(t) = 3\sin 2\pi t$
(ii) $x_2(t) = 5\sin(4\pi t + \pi/6)$
(iii) $x_3(t) = \sin^2(4\pi t)$

(i) Given :

$A \sin \omega t$

$$x_1(t) = 3\sin 2\pi t \quad \text{— (1)}$$

To find, FTP, $T = ?$

Solution :

From eqn (1) $A = 3$; $\omega = 2\pi$

W.K.T $T = \frac{2\pi}{\omega}$

$$T = \frac{\cancel{2\pi}}{\cancel{2\pi}} = 1$$

$T = 1$ Sec.

(ii) Given :

$A \sin(\omega t + \theta)$

$$x_2(t) = 5\sin(4\pi t + \pi/6) \quad \text{— (2)}$$

To find, FTP, $T = ?$

Solution :-

From eqn (2) $A = 5$, $\omega = 4\pi$

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w.k.T $T = \frac{2\pi}{\omega}$

$T = \frac{\cancel{2\pi}}{_2\cancel{4\pi}} = \frac{1}{2}$

$T = 0.5$ Sec.

Given:-

(iii) $x_3(t) = \sin^2(4\pi t)$

$\because \sin^2\theta = \frac{1-\cos 2\theta}{2}$

$x_3(t) = \frac{1-\cos 8\pi t}{2}$

$x_3(t) = \frac{1}{2} - \frac{\cos 8\pi t}{2}$ —— (3)

To find

FTP, $T = ?$

Solution:-

from eqn (3) $\omega = 8\pi$

w.k.T $T = \frac{2\pi}{\omega}$

$T = \frac{\cancel{2\pi}}{\cancel{8}_4\pi} = \frac{1}{4}$

$T = \frac{1}{4}$ Sec.

NOTE:-

The sum of two (or) more than ~~one~~ two periodic signal will be periodic if ratios of their FTP are rational number ie,

$$x(t) = x_1(t) + x_2(t)$$
$$\downarrow \qquad \downarrow$$
$$T_1 \qquad T_2$$

$$T = T_1/T_2 = \text{Rational number} \ldots$$

(2) problem :-

Find FTP of if it is periodic

(i) $x_1(t) = \sin 2t + \cos 3\pi t$
(ii) $x_2(t) = \sin 2\pi t + \cos \sqrt{2}\pi t$
(iii) $x_3(t) = \sin 4\pi t + \sin 7\pi t$.

(i) Given :- $\sin \omega_1 t + \cos \omega_2 t$

$$x_1(t) = \sin 2t + \cos 3\pi t \quad — (1)$$

To find :

Periodic (or) Not.

Solution :-

From eqn (1) $\omega_1 = 2$; $\omega_2 = 3\pi$

W.K.T $T_1 = \frac{2\pi}{\omega_1}$ & $T_2 = \frac{2\pi}{\omega_2}$

$T_1 = 2\pi/2$; $T_2 = 2\pi/3\pi$

$T_1 = \pi$; $T_2 = 2/3$

$$T = T_1/T_2 = \pi/(2/3) = 3\pi/2$$

$$T = 3\pi/2 = \text{Irrational No.}$$

$\therefore$ The given signal $x(t) = \sin 2t + \cos 3$ is a Non periodic signal.

(ii) Given :-

$$x_2(t) = \overset{\sin \omega_1 t}{\sin 2\pi t} + \overset{\cos \omega_2 t}{\cos \sqrt{2}\,\pi t} \quad \text{--- (2)}$$

To find :

Periodic (or) Not.

Solution :-

From eqn (2) $\omega_1 = 2\pi$; $\omega_2 = \sqrt{2}\pi$

w.k.r

$$T_1 = \frac{2\pi}{\omega_1} \quad ; \quad T_2 = 2\pi/\omega_2$$

$$T_1 = 2\pi/2\pi \quad ; \quad T_2 = 2\pi/\sqrt{2}\pi$$

$$T_1 = 1 \quad ; \quad T_2 = 2/\sqrt{2} = \sqrt{2}$$

$$T = T_1/T_2 = 1/\sqrt{2} = \text{Irrational No}$$

$\therefore$ The given signal $x(t) = \sin 2\pi t + \cos \sqrt{2}\pi t$ is a Non periodic signal.

(iii) Given :-

$$x_3(t) = \overset{\sin \omega_1 t}{\sin 4\pi t} + \overset{\sin \omega_2 t}{\sin 7\pi t} \quad \text{--- (3)}$$

To find :

Periodic or NOT.

Solution :-

from eqn (3) $\omega_1 = 4\pi$; $\omega_2 = 7\pi$

W.K.T $T_1 = 2\pi/\omega_1$; $T_2 = 2\pi/\omega_2$

$\therefore T_1 = 2\pi/4\pi$; $T_2 = 2\pi/7\pi$

$T_1 = 1/2$; $T_2 = 2/7$

$$T = \frac{T_1}{T_2} = \frac{1/2}{2/7} = \frac{1}{2} \times \frac{7}{2} = \frac{7}{4}$$

= Rational No

$\therefore$ The given signal $x_3(t) = \sin 4\pi t + \sin 7\pi t$ is a periodic signal.

ENERGY and POWER signals :-

An energy signal is one which has finite energy and zero power. Hence $x(t)$ is an energy signal if :

$$\boxed{0 < E < \infty \; ; \; P = 0}$$

The power signal is one which has finite power and infinite energy, Hence $x(t)$ is a power signal if :-

$$\boxed{0 < P < \infty \; ; \; E = \infty}$$

For CT signal, $x(t)$

$$\text{Energy}, E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

$$\text{Power}, P = \underset{T \to \infty}{Lt} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt$$

For DT signal, $x(n)$:

$$\text{Energy}, E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$\text{Power}, P = \underset{N \to \infty}{Lt} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|$$

NOTE :-

* Non periodic signals are energy Signal

eg : A single rectangular pulse

* periodic signals are power Signal

eg : periodic pulse signal.

Sigularity and Related Signals :-

Singularity signals are those for which the left and right limits are not same, ie. sigularity signals are discontinuous at a particular point (or) point like, Impulse signal, step signal, Signum signal and Ramp signal.

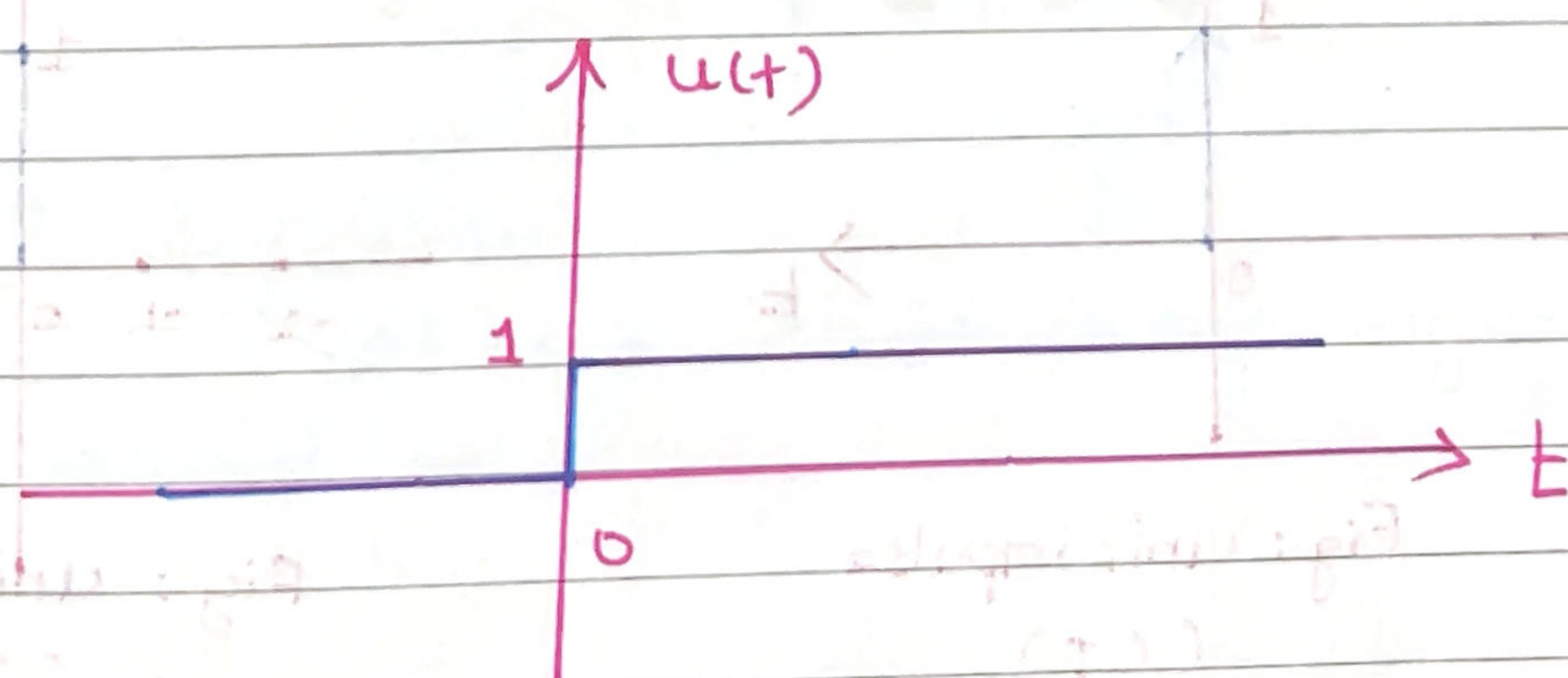
Unit step, unit Impulse, unit ramp, pulse, real and complex exponential, sinusoidal signals are known as standards signals (or) Test signals (or) Basic elementary signal.

UNIT STEP SIGNALS :-

Unit step signals can be represented as $u(t)$ and defined as,

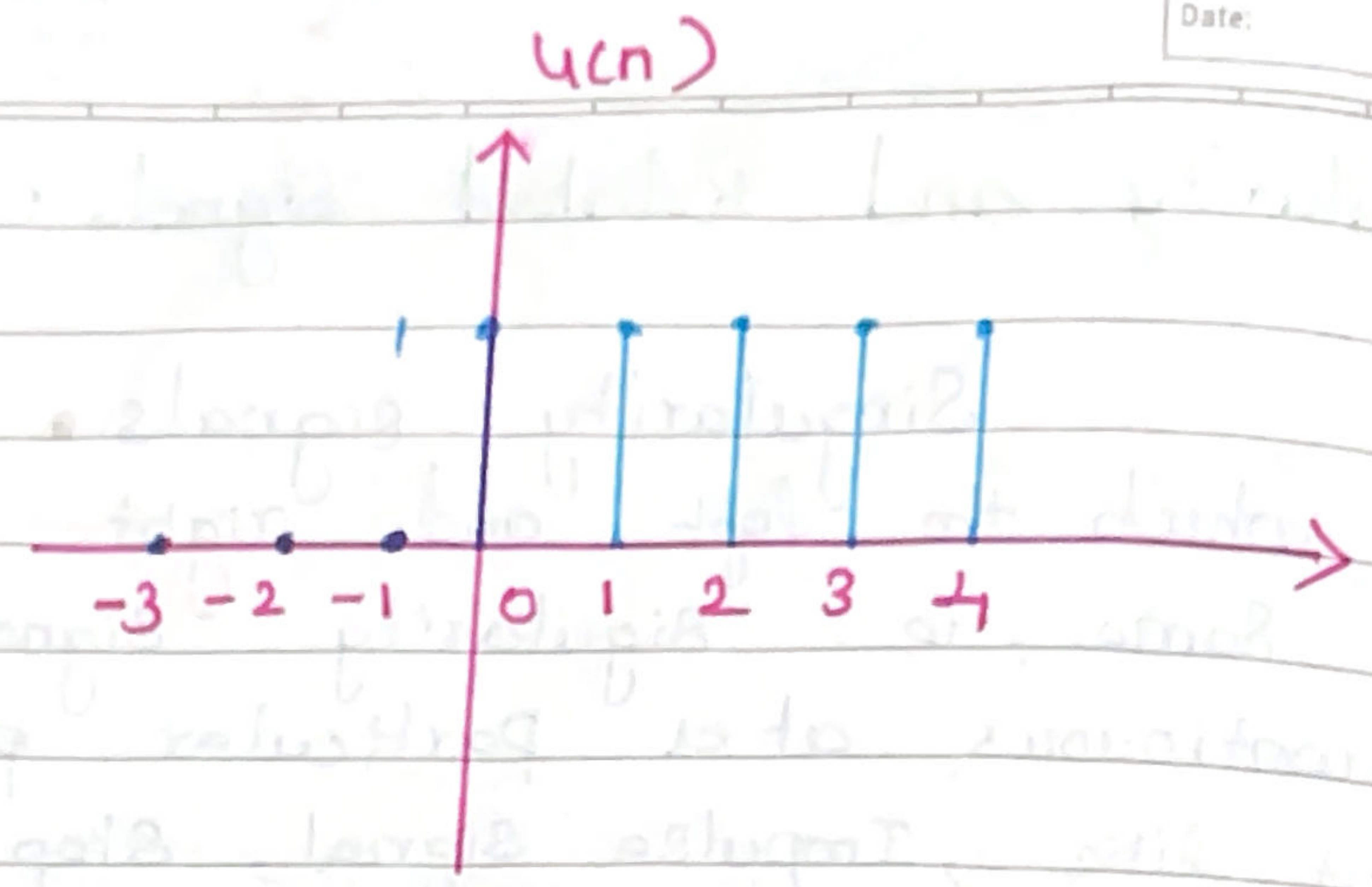
For CT,

$$u(t) = \begin{cases} 1 & ; \; t \geq 0 \\ 0 & ; \; t < 0 \end{cases}$$



DT unit step signal can be represented as $u(n)$ and defined as,

$$u(n) = \begin{cases} 1 & ; \; n \geq 0 \\ 0 & ; \; n < 0 \end{cases}$$



UNIT IMPULSE SIGNAL:-

Unit Impulse Signal can be represented as $\delta(t)$ and defined as, for CT:-

$$\delta(t) = \begin{cases} 1 ; & t = 0 \\ 0 ; & t \neq 0 \end{cases}$$

Dr unit impulse signal can be represented as $\delta(n)$ and defined as,

$$\delta(n) = \begin{cases} 1 ; & n = 0 \\ 0 ; & n \neq 0 \end{cases}$$

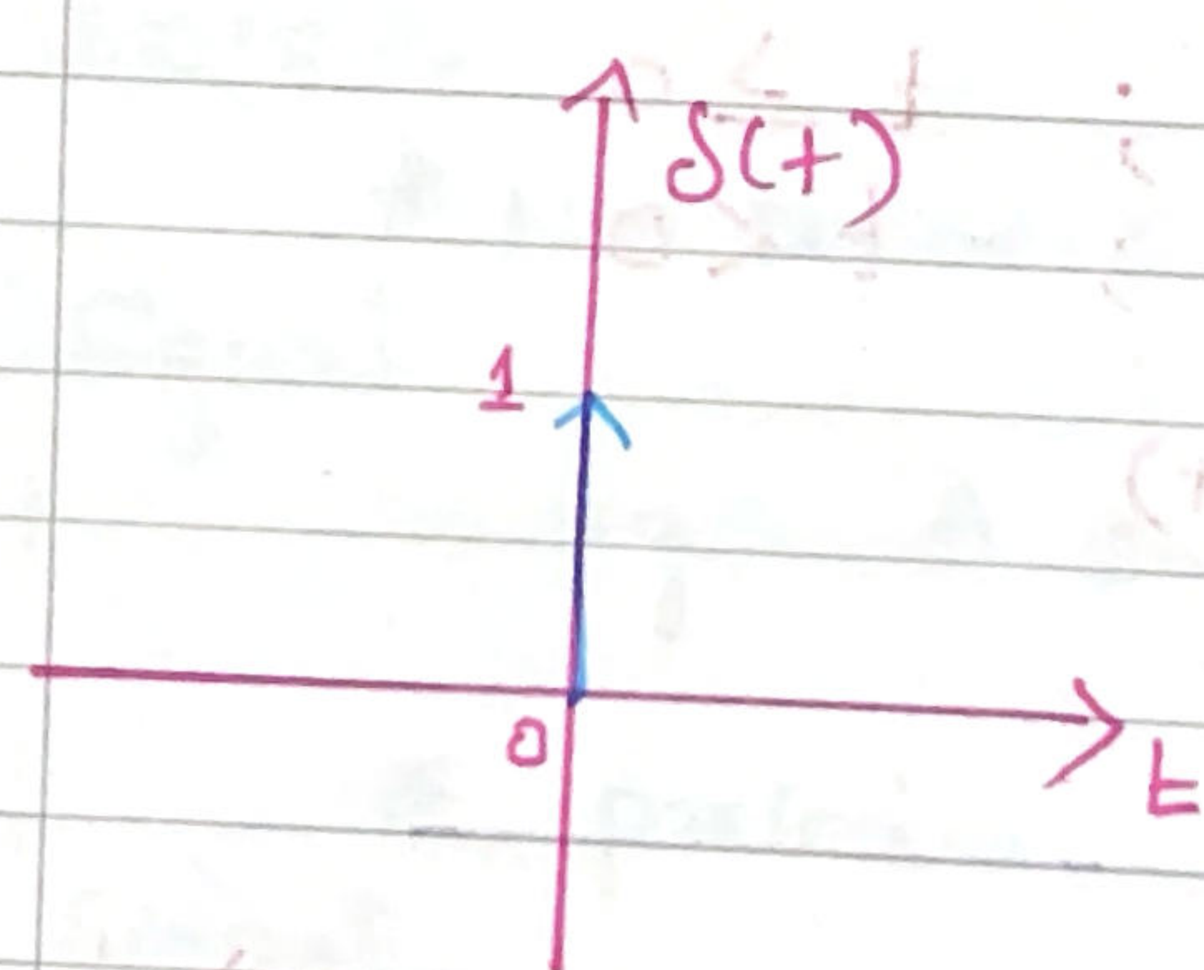


Fig: unit impulse (CT)

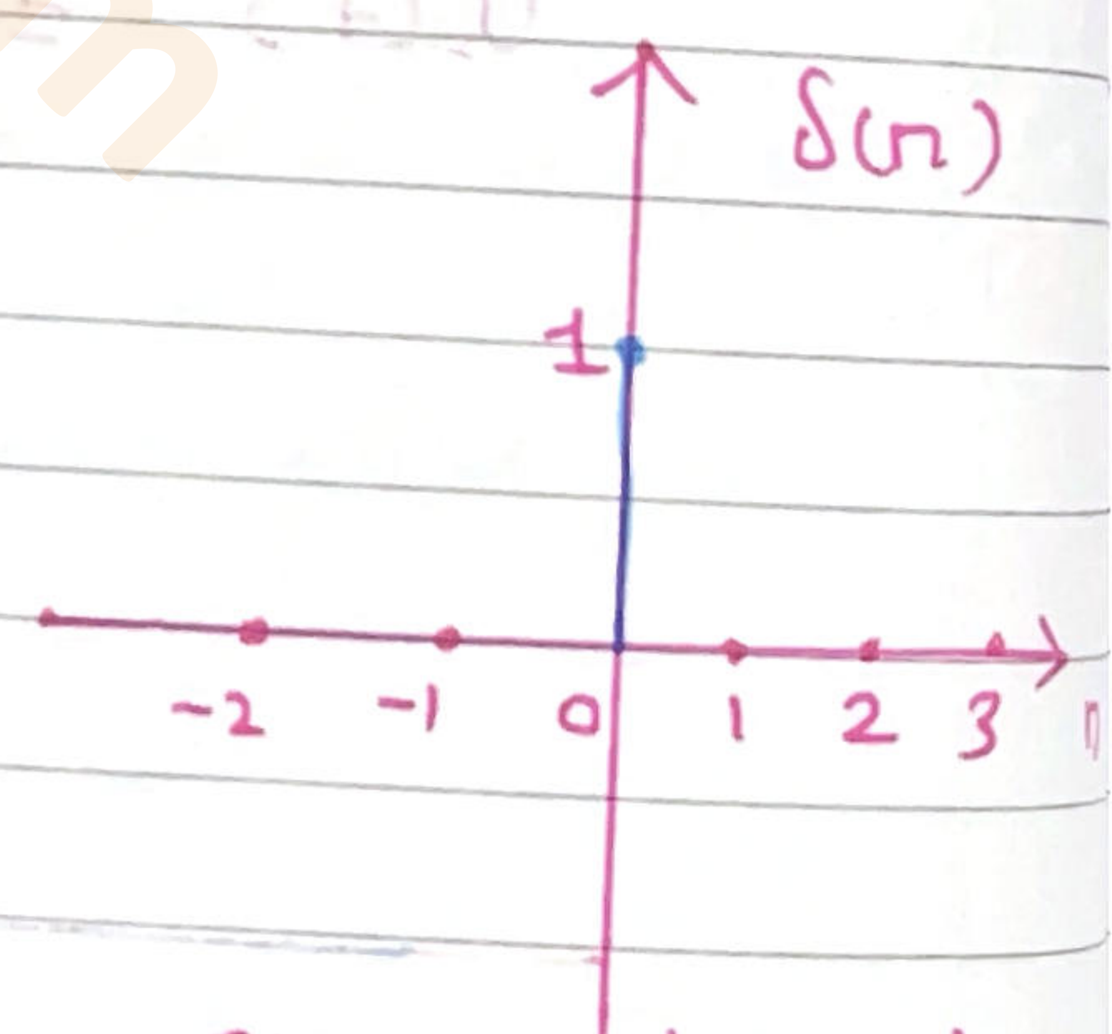
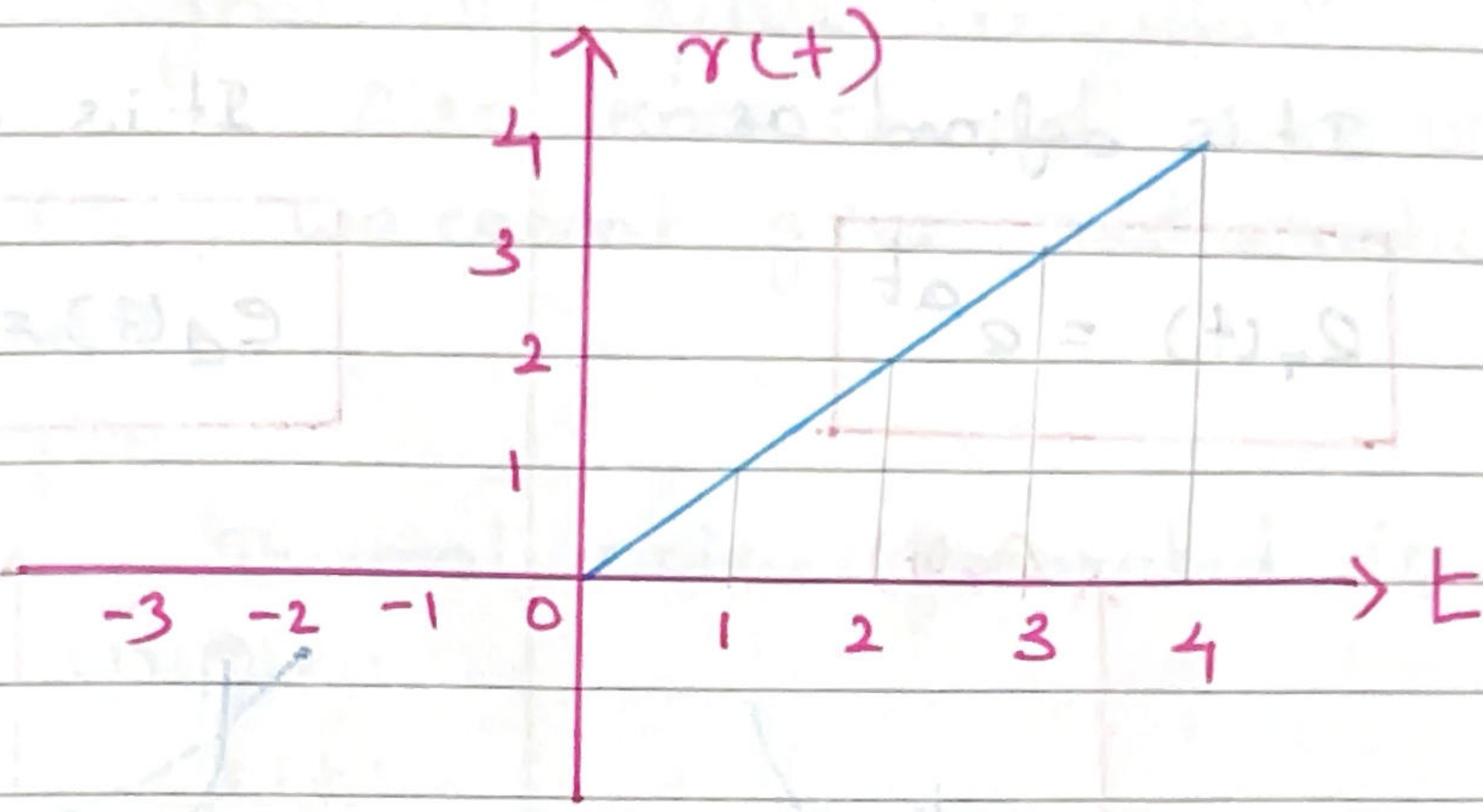


Fig: unit impulse (DT)

UNIT RAMP SIGNAL:

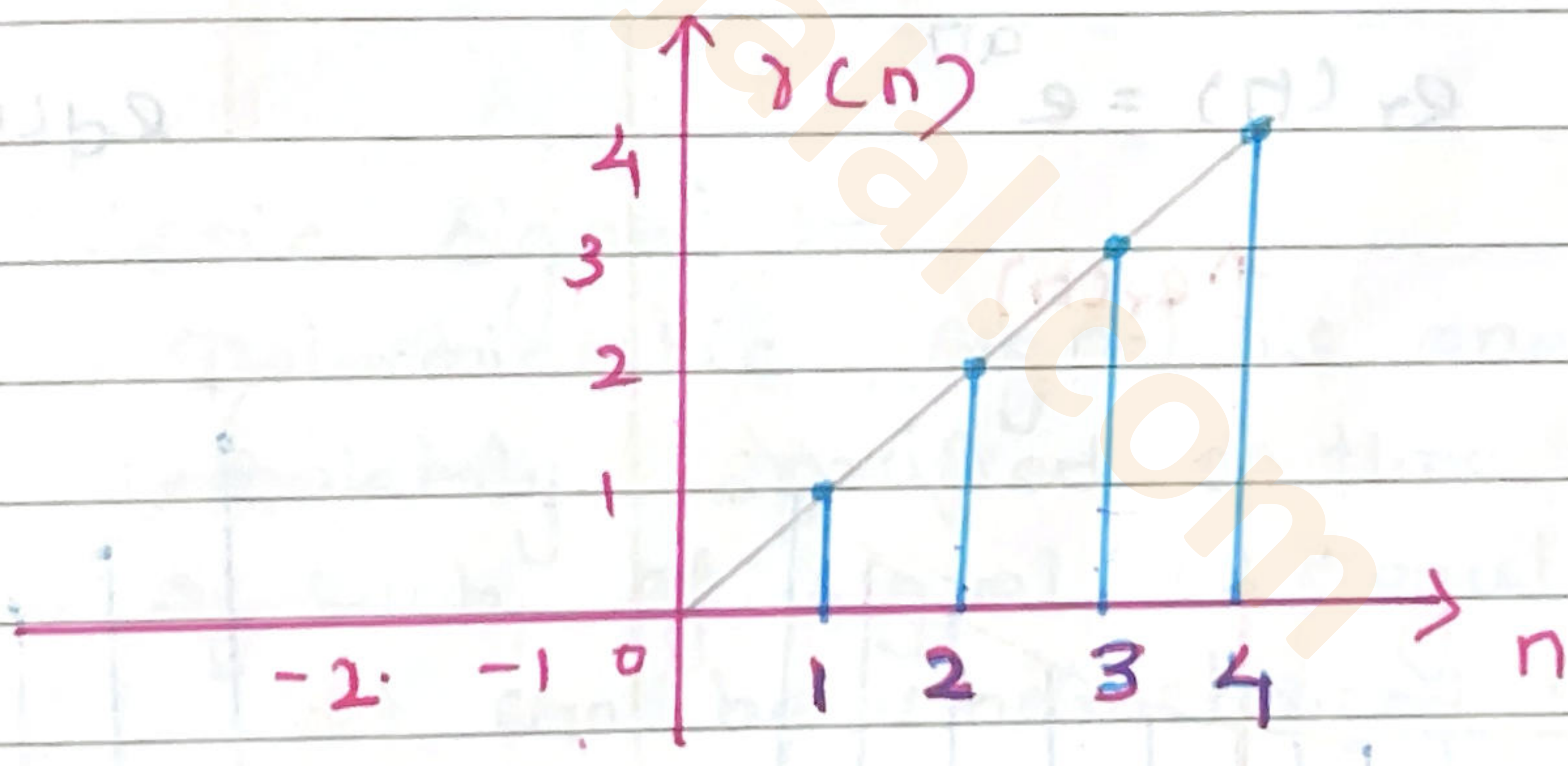
The unit ramp signal can be represented as $r(t)$ and defined as.

$$r(t) = \begin{cases} t & ; t \geq 0 \\ 0 & ; t < 0 \end{cases}$$



The DT unit ramp signal can be represented as $r(n)$ and defined as

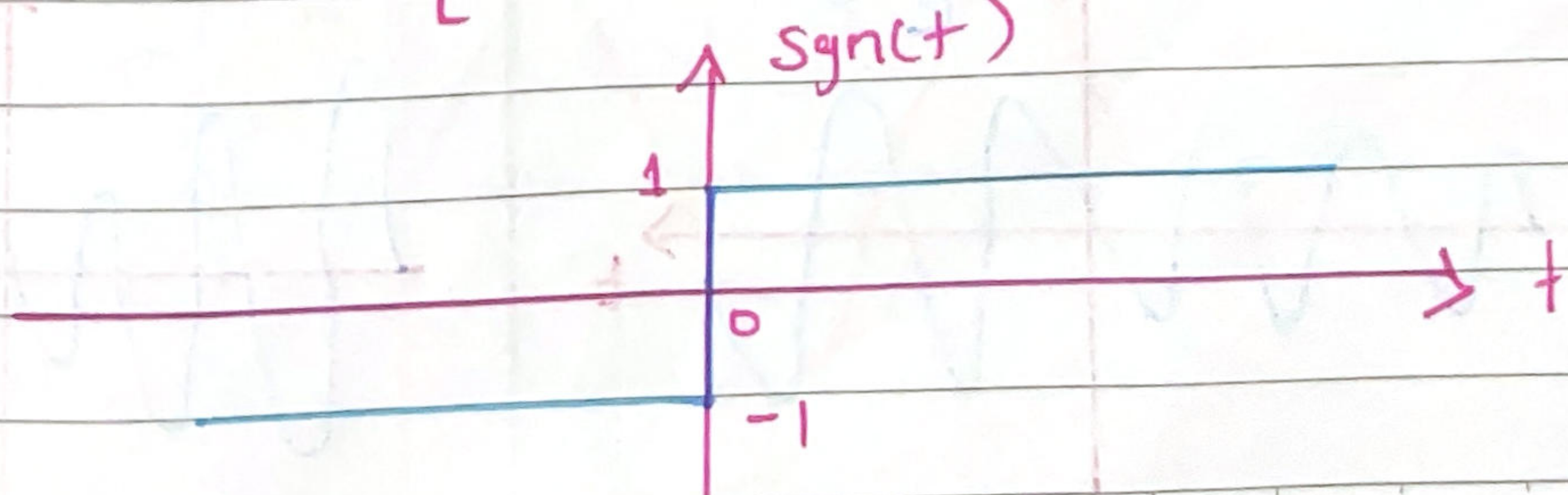
$$r(n) = \begin{cases} n & ; n \geq 0 \\ 0 & ; n < 0 \end{cases}$$



Signum signal:-

It is a double step signal and it is denoted with $\text{Sgn}(t)$

$$\text{Sgn}(t) = \begin{cases} 1 & ; t > 0 \\ -1 & ; t < 0 \end{cases}$$

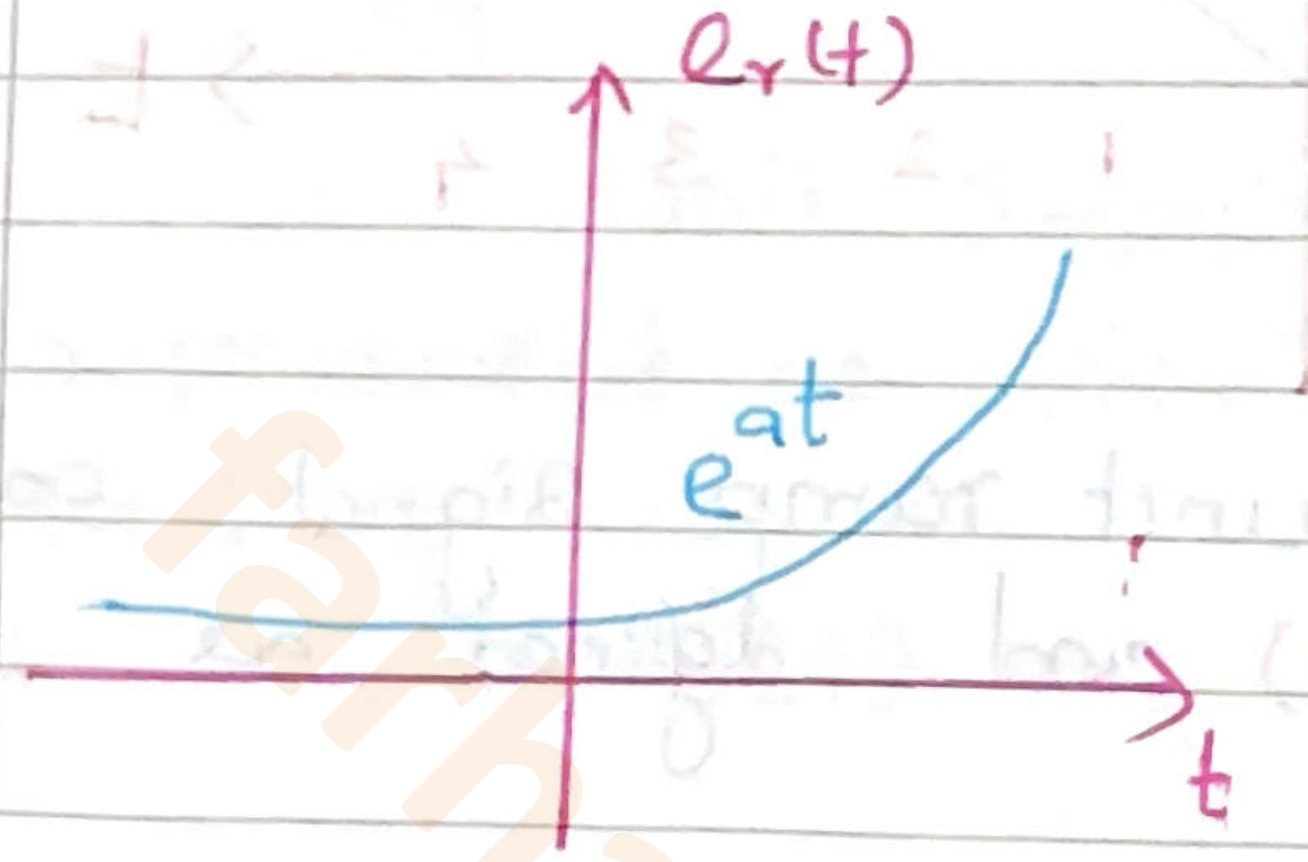


REAL EXPONENTIAL SIGNALS :-

Rising Real Exponential

For CT:- It is defined as

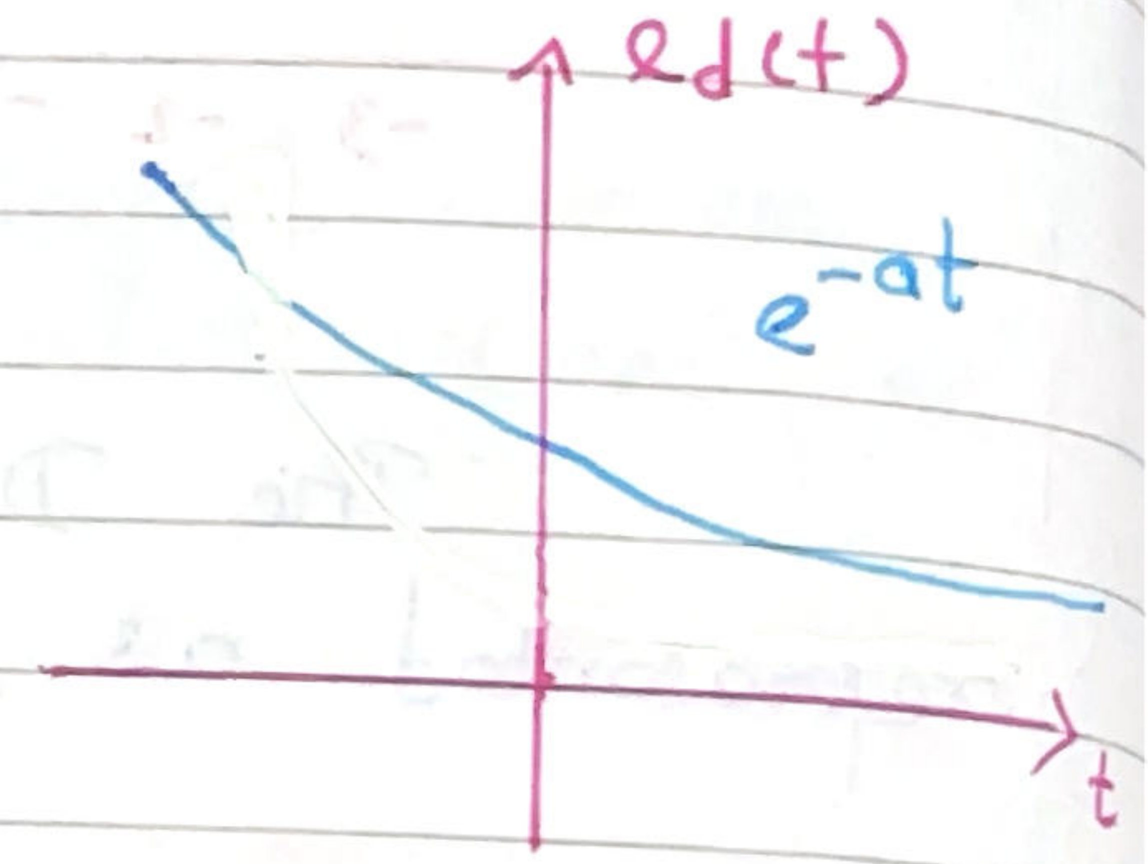
$$\boxed{e_r(t) = e^{at}}$$



Decaying Real exponential

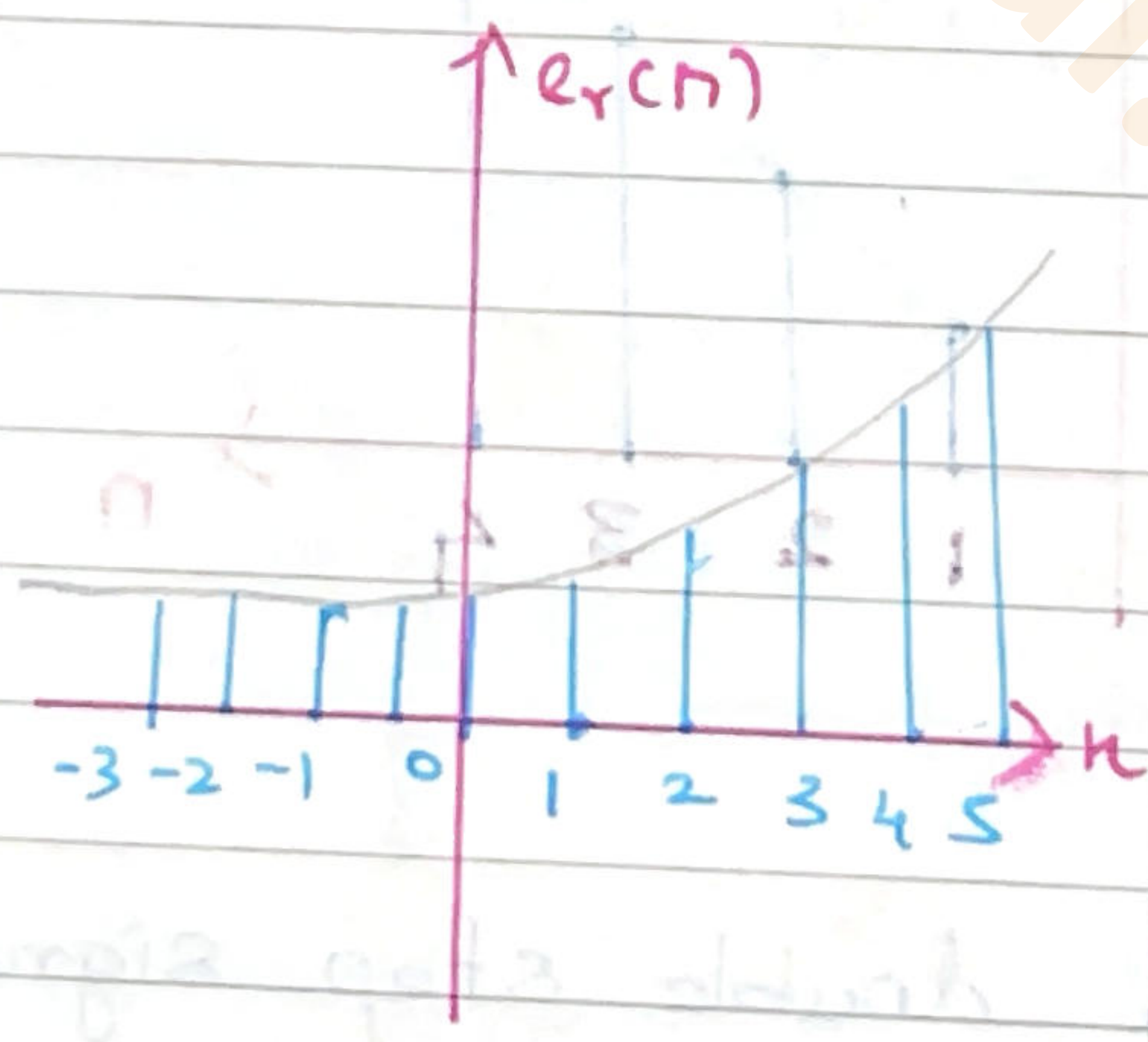
It is defined as

$$\boxed{e_d(t) = e^{-at}}$$



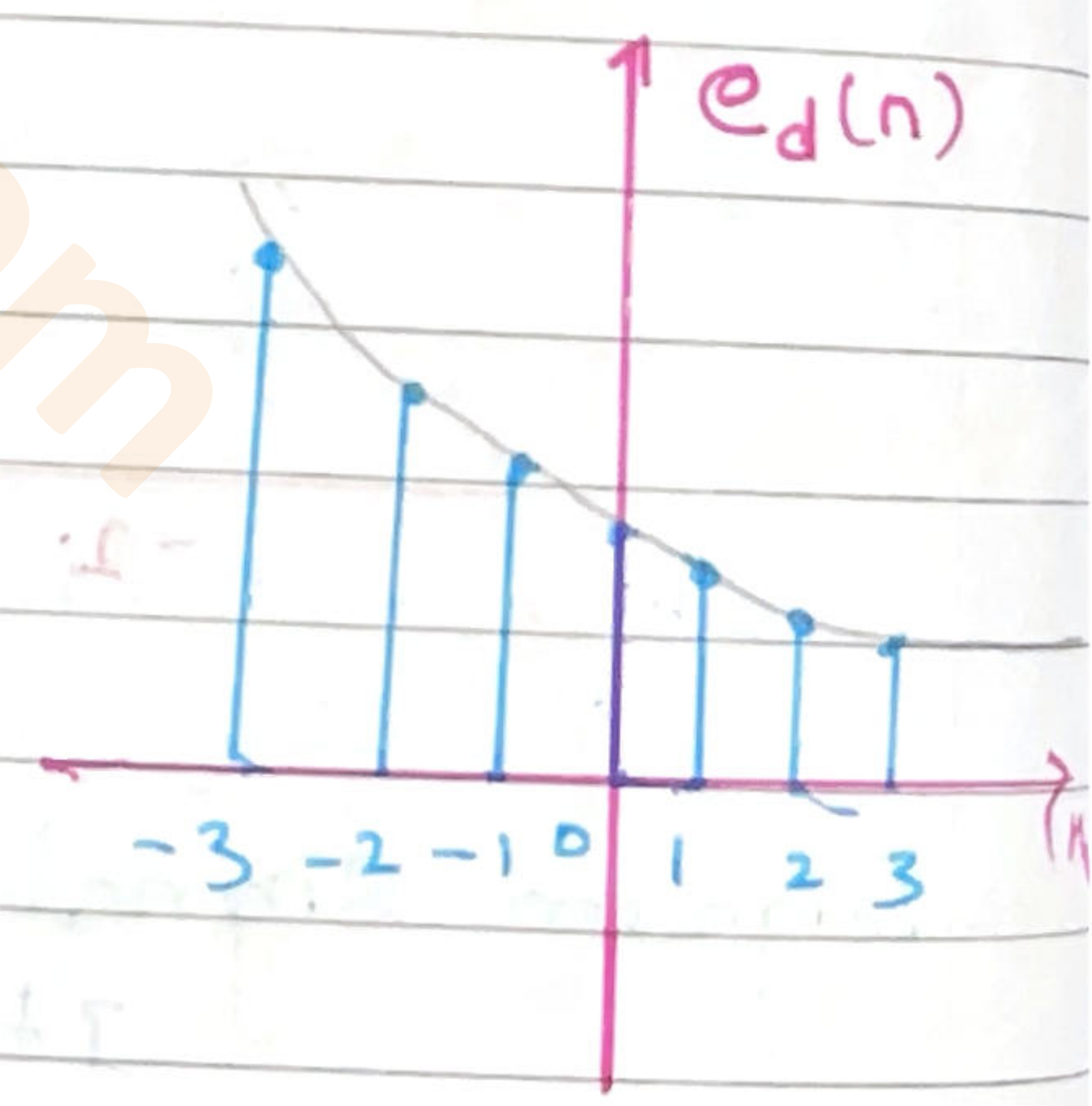
For DT: It is defined as

$$e_r(n) = e^{an}$$



It is defined as

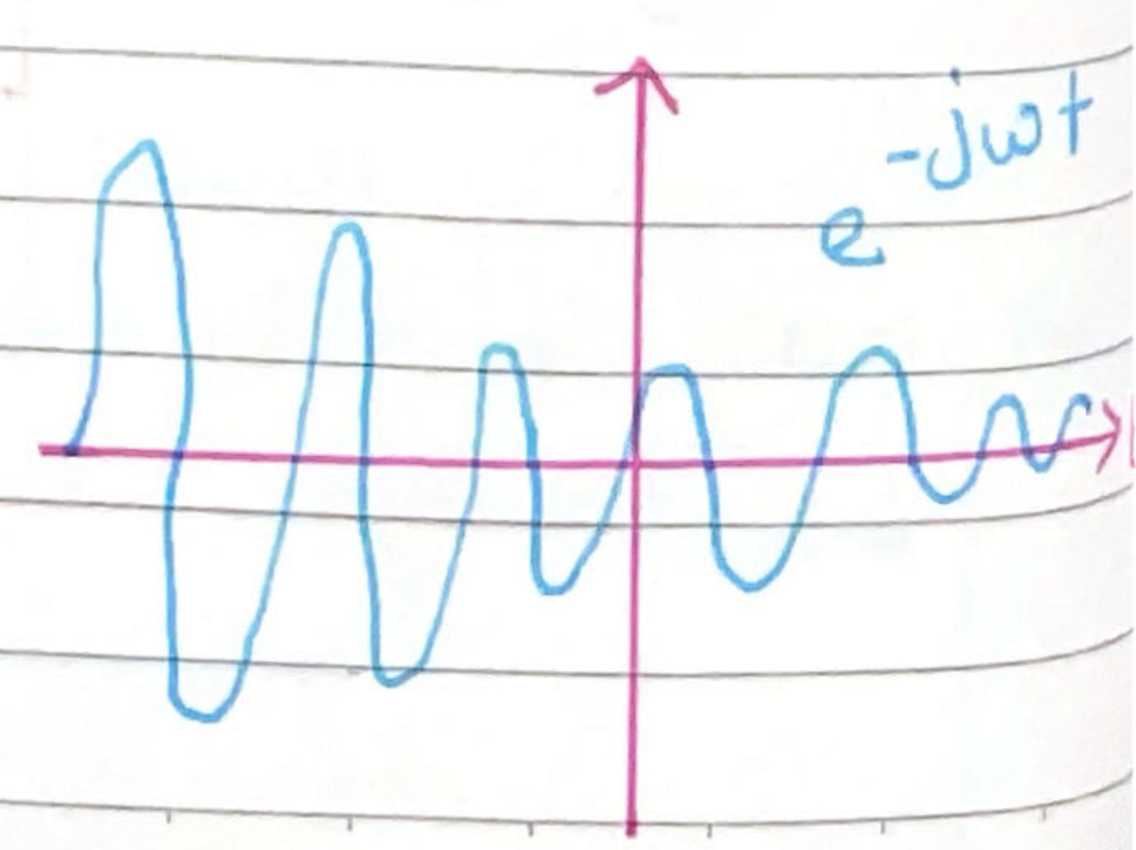
$$e_d(n) = e^{-an}$$



Complex Exponential Signals :-

Rising Complex exponential

Decaying complex exponential



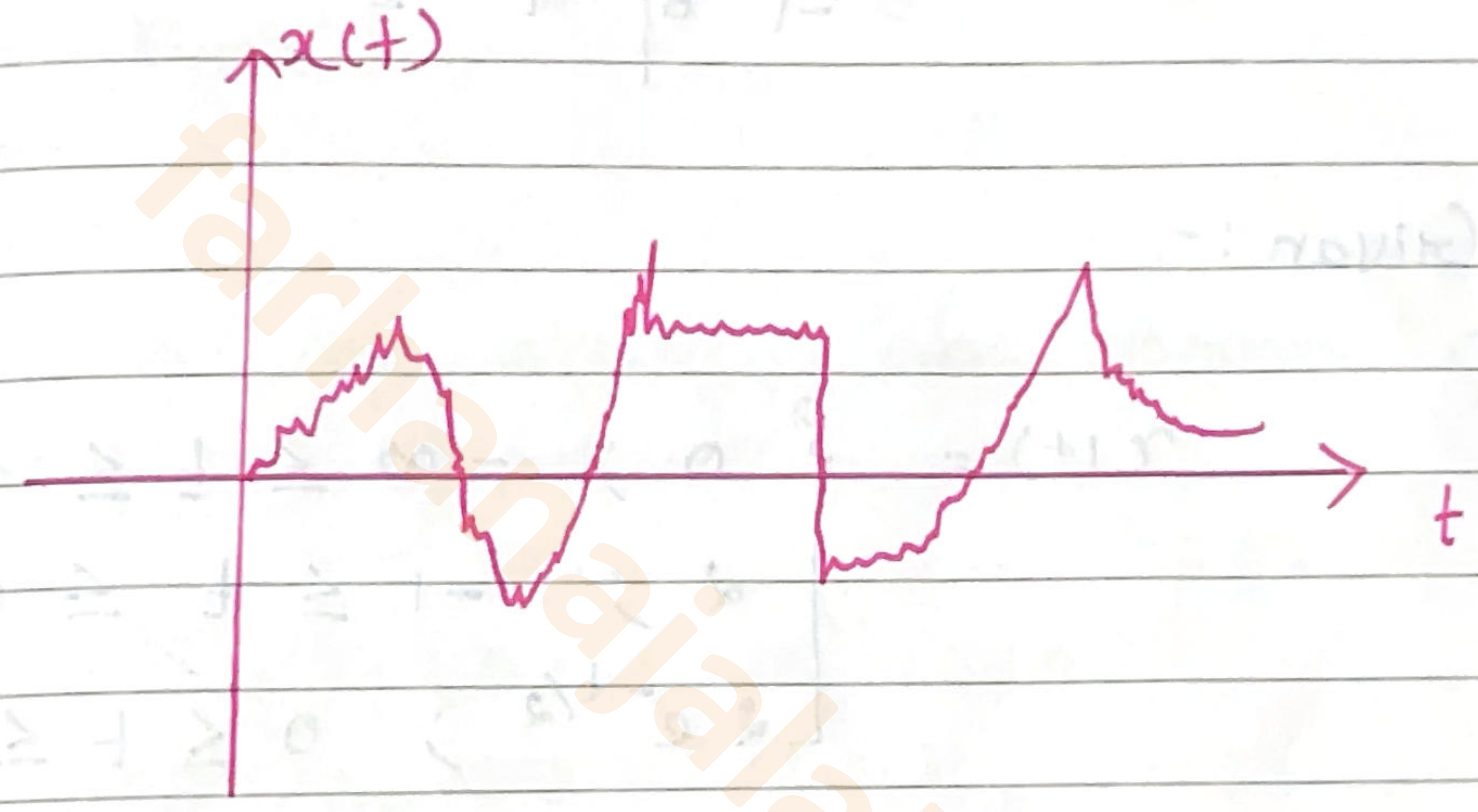
Random Signal:-

Random signal is one whose occurance is always in nature. The pattern of such signal is quite irregular.

Also known as NON DETERMINISTIC signal (ie, we cannot give mathematical discription).

Eg:-

Thermal noise generated in an electric circuit.



Deterministic Signal:-

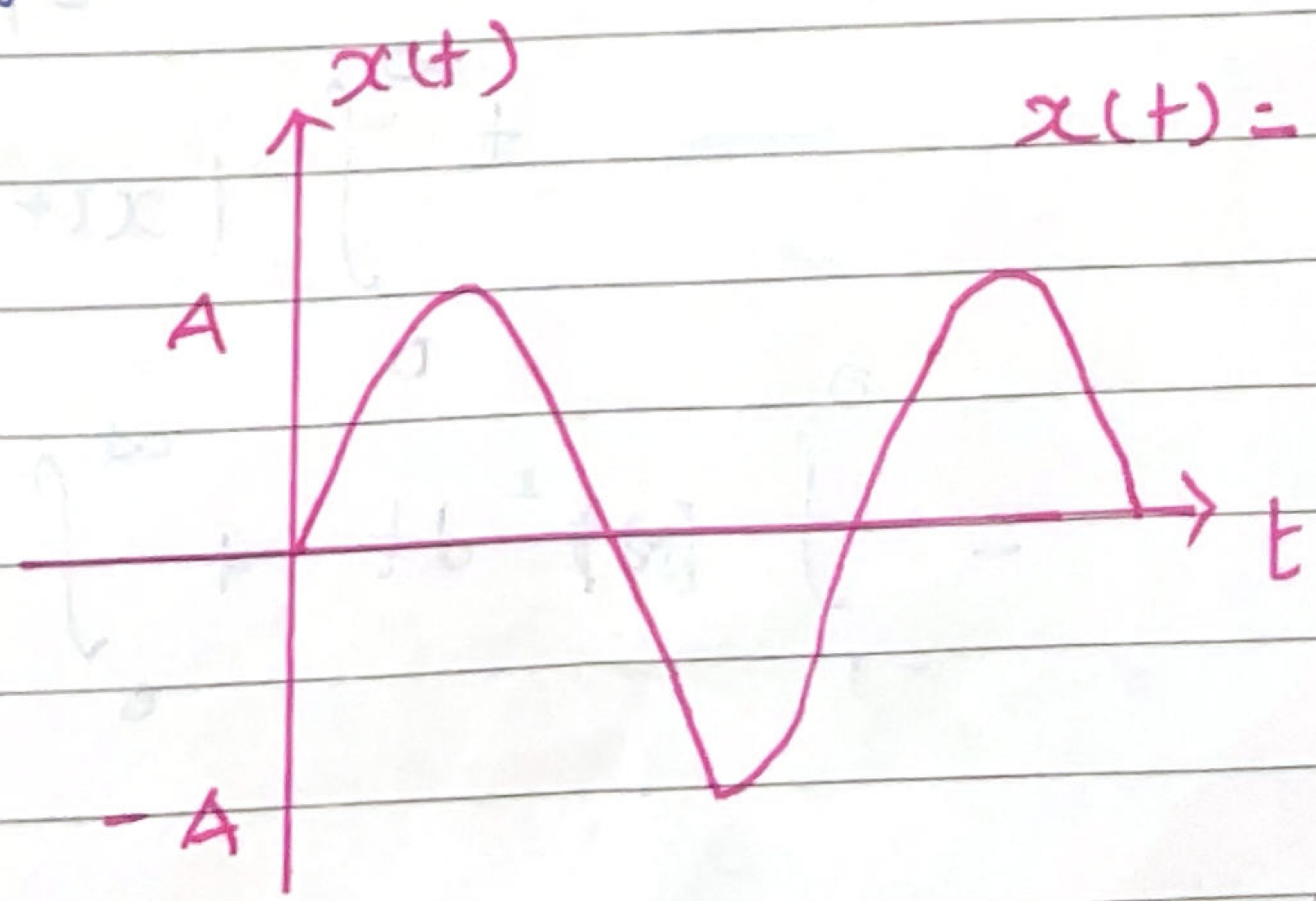
Deterministic signal is one which can be completely specified in time. The parttern of such of signal is regular.

It can be characterized (or) defined mathematically.

Also, the nature and amplitude of such a signal at any time can be predicted.

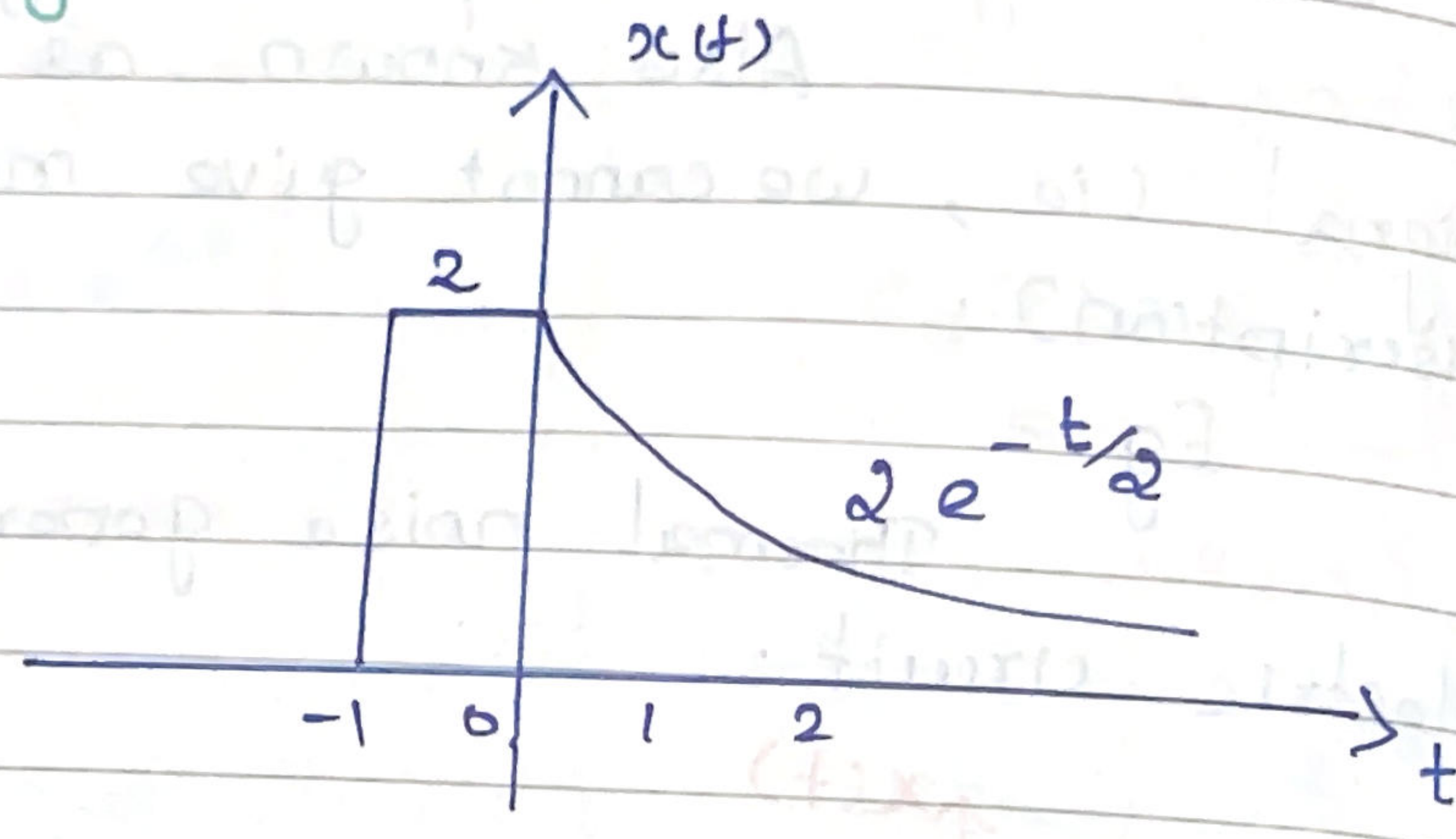
Eg: sinusoidal signal.

$x(t) = A\sin\omega t$



1. Problem :-

Fig shows the signal $x(t)$ Determine whether the signal is an energy (or) a power signal (or) neither.



Given :-

$$x(t) = \begin{cases} 0 \; ; & -\infty \leq t \leq -1 \\ 2 \; ; & -1 \leq t \leq 0 \\ 2e^{-t/2} \; ; & 0 \leq t \leq \infty \end{cases}$$

To Find :

Energy E & power P

Solution :-

W.K.T Energy $E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$

$$E = \int_{-\infty}^{-1} |x(t)|^2 \, dt + \int_{-1}^{0} |x(t)|^2 \, dt + \int_{0}^{\infty} |x(t)|^2 \, dt$$

$$= \int_{-1}^{0} (2)^2 \, dt + \int_{0}^{\infty} |2e^{-t/2}|^2 \, dt$$

$$= 4\left[t\right]_{-1}^{0} + 4\left[-e^{-t}\right]_{0}^{\infty}$$

$$= 4(0-(-1)] + 4\left[-e^{-\infty} - (-e^{0})\right]$$

$$= 4 + 4 = 8$$

Energy, $E = 8$ $\quad \therefore$ power $= 0$

$\therefore$ The given signal $x(t)$ is an energy signal.

(2) problem :-

Find the energy and power of following sequences

(i) $x_1(n) = a^n u(n)$

(ii) $x_2(n) = u(n)$

(i) Given :

$$x_1(n) = a^n u(n)$$

$\because$ $u(n)$ = unit step signal = $\begin{cases} 1 ; & n \geq 0 \\ 0 ; & n < 0 \end{cases}$

To find :-

Energy, E & power, P.

Solution :-

W.K.T $\quad E = \sum_{n=-\infty}^{\infty} |x(n)|^2$

$$= \sum_{n=-\infty}^{\infty} |a^n u(n)|^2$$

$$= \sum_{n=0}^{\infty} |a^n|^2$$

$$= \sum_{n=0}^{\infty} a^{2n}$$

$$= \sum_{n=0}^{\infty} (a^2)^n$$

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a} \quad \text{if } a \neq 1$$

$$E = \frac{1}{1-a^2} \quad ; \quad \text{Power } P = 0$$

$\therefore$ The signal $x_1(n)$ is an energy signal.

(ii) Given :

$$x_2(n) = u(n)$$

$$u(n) = \begin{cases} 1 ; & n \geq 0 \\ 0 ; & n < 0 \end{cases}$$

To find :

Energy $E = ?$; Power $P = ?$

Solution :

$$\text{Energy} : E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$= \sum_{n=-\infty}^{\infty} |u(n)|^2$$

$$= \sum_{n=-\infty}^{-1} |u(n)|^2 + \sum_{n=0}^{\infty} |u(n)|^2$$

$$= \sum_{n=0}^{\infty} (1)^2$$

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$$E = \infty$$

$$\text{power} : p = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |u(n)|^2$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} \left\{ \sum_{n=-N}^{-1} |u(n)|^2 + \sum_{n=0}^{N} |u(n)|^2 \right\}$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=0}^{N} (1)^2$$

$$\because \sum_{n=0}^{N} (1)^n = N+1$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} N+1$$

$$= \lim_{N \to \infty} \frac{N(1+1/N)}{2N(1+1/N)}$$

$$= \frac{1+1/\infty}{2(1+1/\infty)} \qquad \because 1/\infty = 0$$

$$P = 1/2$$

$\therefore$ The given signal $x_2(n) = u(n)$ is a power signal.

(3) Problem :-

Find energy and power of ramp signal :-

Given :-

Unit Ramp Signal $r(t) = \begin{cases} t \,; & t \geq 0 \\ 0 \,; & t < 0 \end{cases}$

To find :

Energy $E = ?$; power $P = ?$

Solution :

$$\text{Energy } E = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$= \int_{-\infty}^{0} 0 + \int_{0}^{\infty} (t)^2 dt$$

$$= \left[\frac{t^3}{3}\right]_0^{\infty} \qquad \because \int a^n = \frac{a^{n+1}}{n+1}$$

$$\boxed{E = \infty}$$

$$\text{Power}, P = \underset{T\to\infty}{Lt} \int_{-T}^{T} |x(t)|^2 dt$$

$$P = \underset{T\to\infty}{Lt} \frac{1}{T} \int_{-T}^{T} |x(t)|^2 dt$$

$$= \underset{T\to\infty}{Lt} \frac{1}{T} \int_{0}^{T} (t)^2 dt$$

$$= \underset{T\to\infty}{Lt} \frac{1}{T} \left[\frac{t^3}{3}\right]_0^{T}$$

$$= \underset{T\to\infty}{Lt} \frac{1}{T} \left[\frac{T^3}{3}\right]$$

$$\boxed{P = \infty}$$

$\therefore$ The given signal, $x(t)$ is either energy nor power.

Operations of Signals :-

Addition, subtraction, Multiplication and division are the few basic operations used in most of the occations. In addition to this basic operations, we have few more which are widely used in signal analysis.

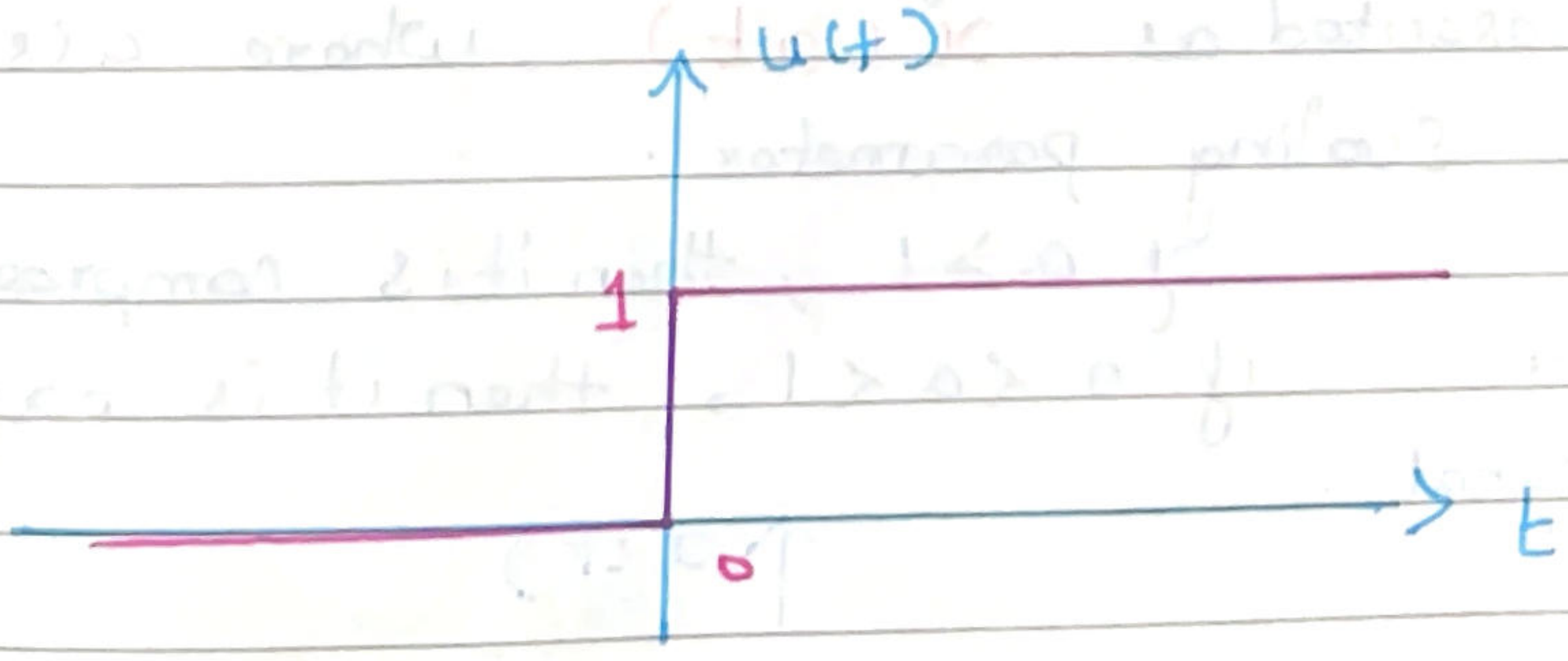
* Time shifting
* Time Scaling
* Time Reversal (or) folding
* Amplitude Shifting
* Amplitude Scaling.

Time shifting operation :-

If time shifting operation is applied on a signal $x(t)$, then the signal is shifted to left (or) right without changing its characteristics. It is represented with $x(t \pm t_0)$, where t_0 is shift

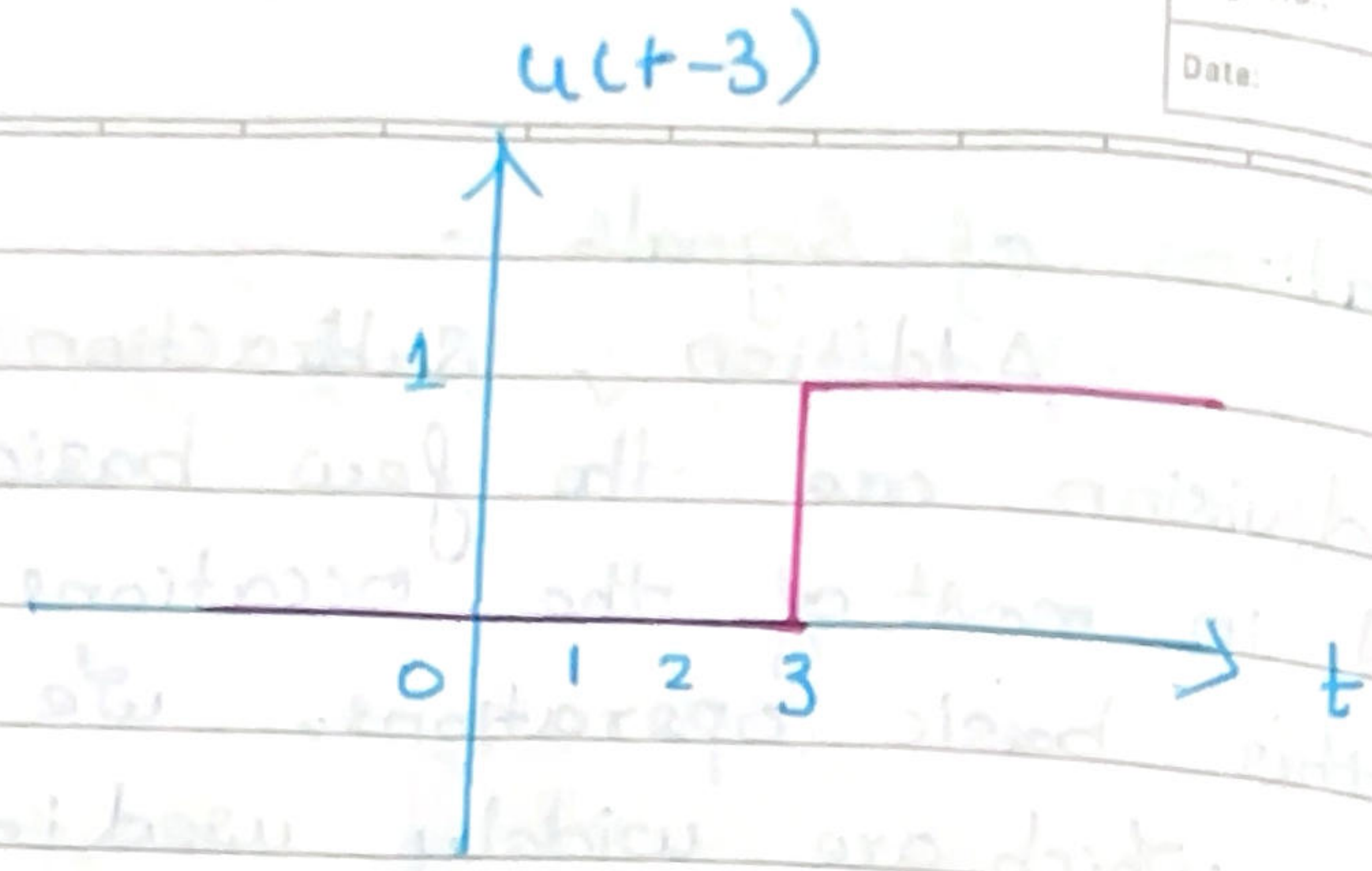
eg :- Given unit step signal

$$u(t) = \begin{cases} 1 \ ; & t \geq 0 \\ 0 \ ; & t < 0 \end{cases}$$



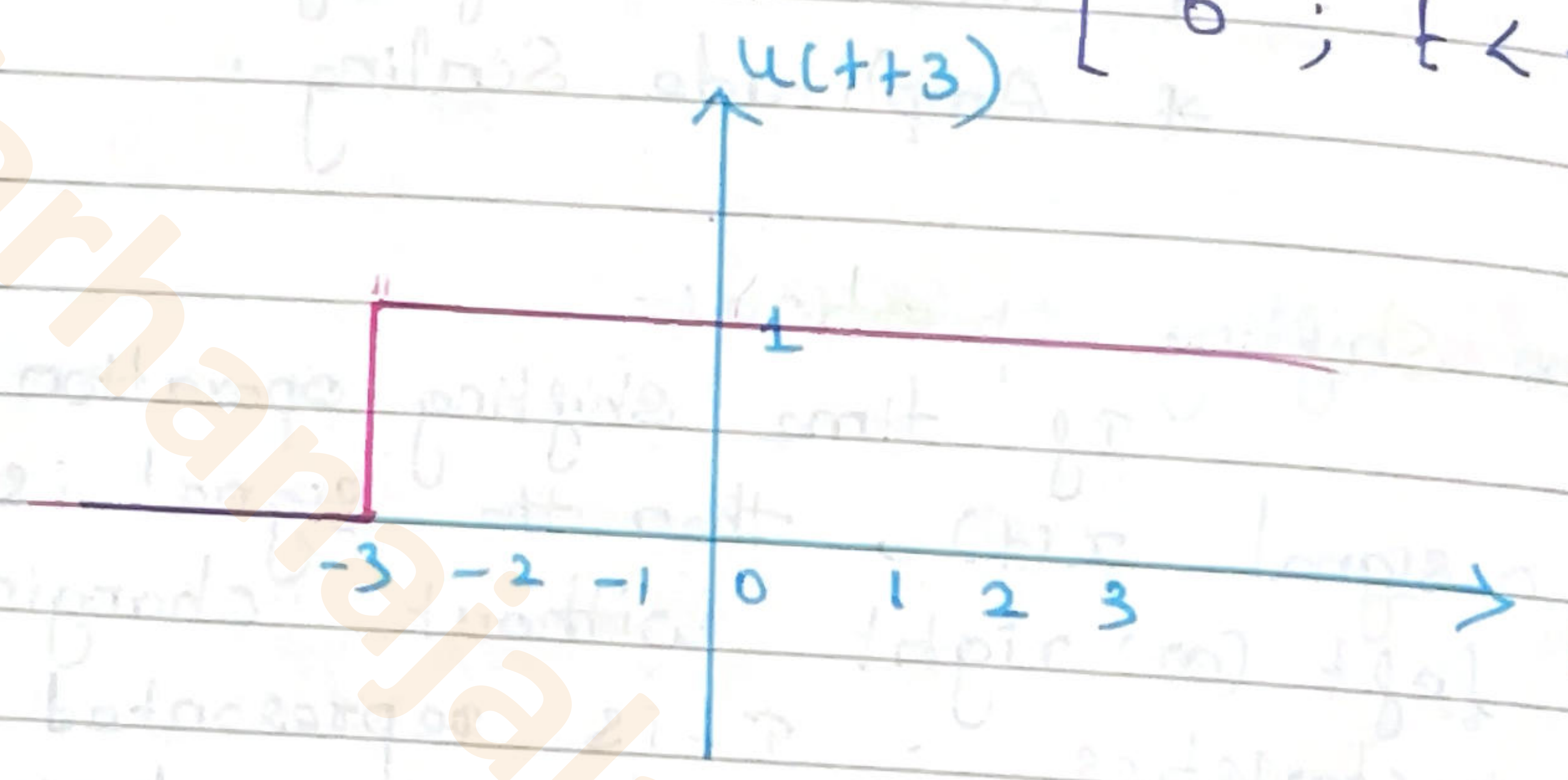
* Shifting $x(t)$ by 3 units to right is represented as $x(t-3)$

$$x(t-3) = u(t-3) = \begin{cases} 1 \ ; & t \geq 3 \\ 0 \ ; & t < 3 \end{cases}$$



* Shifting $x(t)$ by 3 unit left is represented with $x(t+3)$

$$x(t+3) = u(t+3) = \begin{cases} 1 & ; t \geq -3 \\ 0 & ; t < -3 \end{cases}$$



Time scaling operation:-

If the time Scaling operation is applied on a signal $x(t)$, then the signal is compressed (or) expanded in time axis without changing its amplitude. It is represented as $x(at)$, where a is a time scaling parameter.

If $a > 1$, then it is compressed. and if $0 < a < 1$, then it is expanded signal.

Eg:-