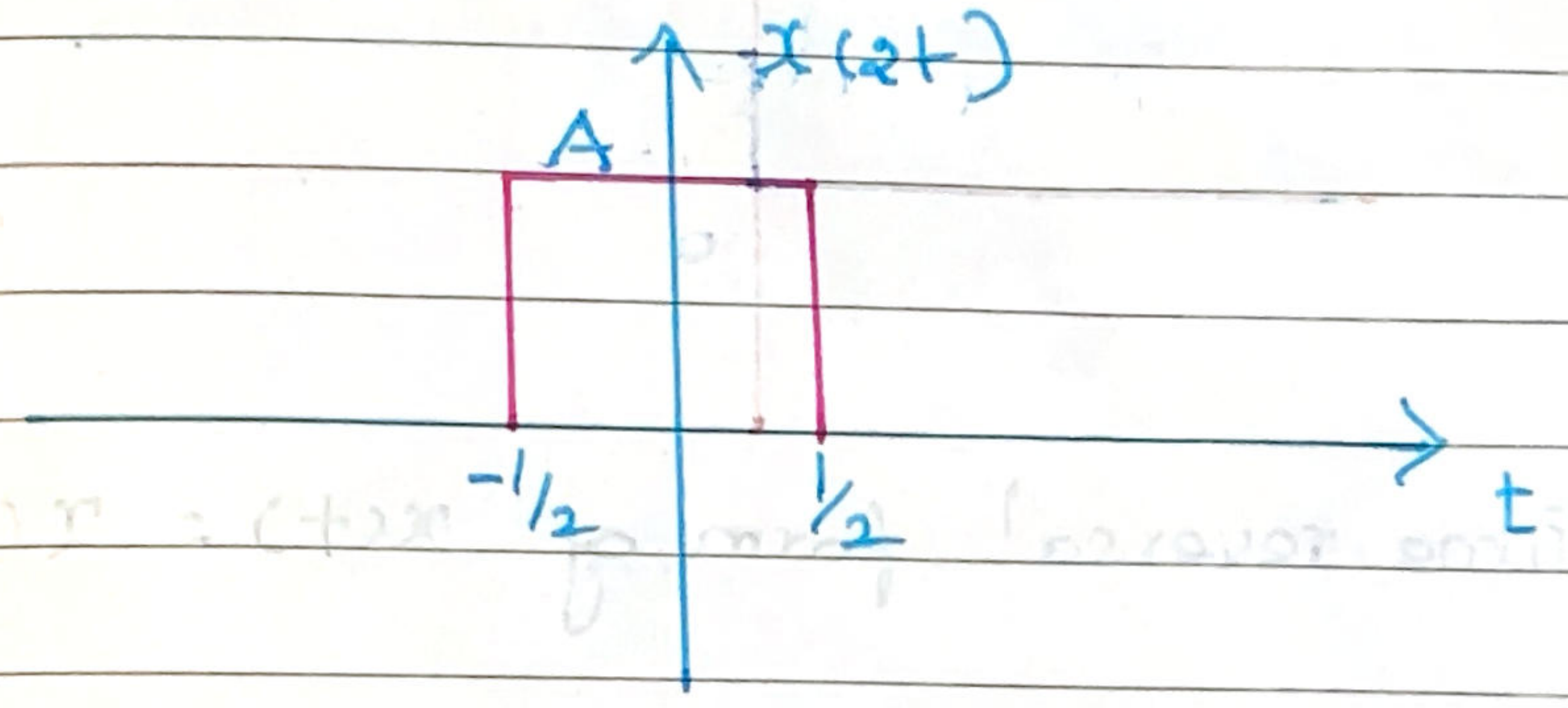
Given Rectangular signal

$$x(t) = A.\,\text{rect}(t/2) = \begin{cases} A, & -1 < t < 1 \\ 0, & \text{otherwise.} \end{cases}$$

* Time scaling (compressed) form of $x(t)$,

$$x(2t) = A.\,\text{rect}\left(\frac{2t}{2}\right) =$$

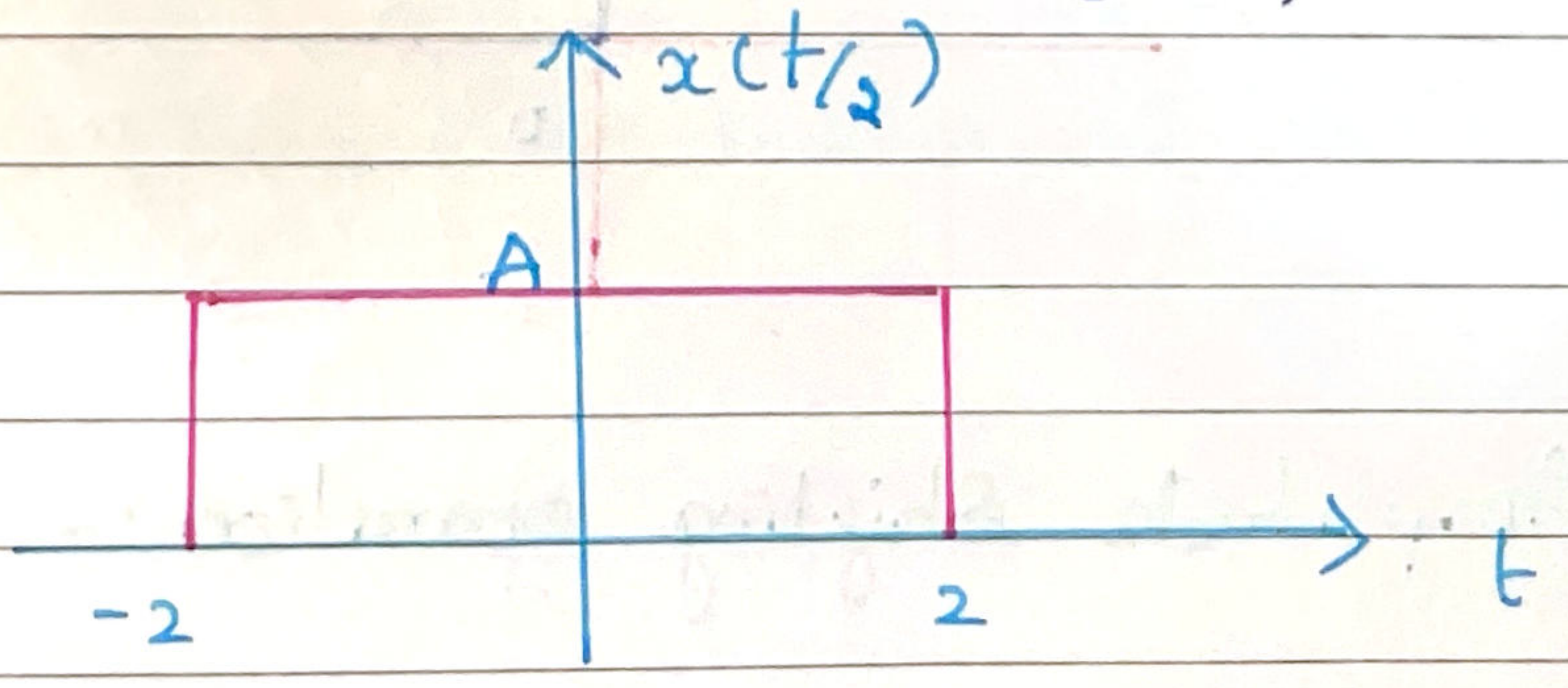
$$A.\,\text{rect}(t/1) = \begin{cases} A, & -1/2 < t < 1/2 \\ 0, & \text{otherwise} \end{cases}$$



* Time Scaling (Expanded) form of $x(t)$,

$$x(t/2) = A.\,\text{rect}\left(\frac{t/2}{2}\right)$$

$$= A.\,\text{rect}(t/4) = \begin{cases} A, & -2 < t < 2 \\ 0, & \text{otherwise} \end{cases}$$



Time Reversal (or) Folding Operation :-

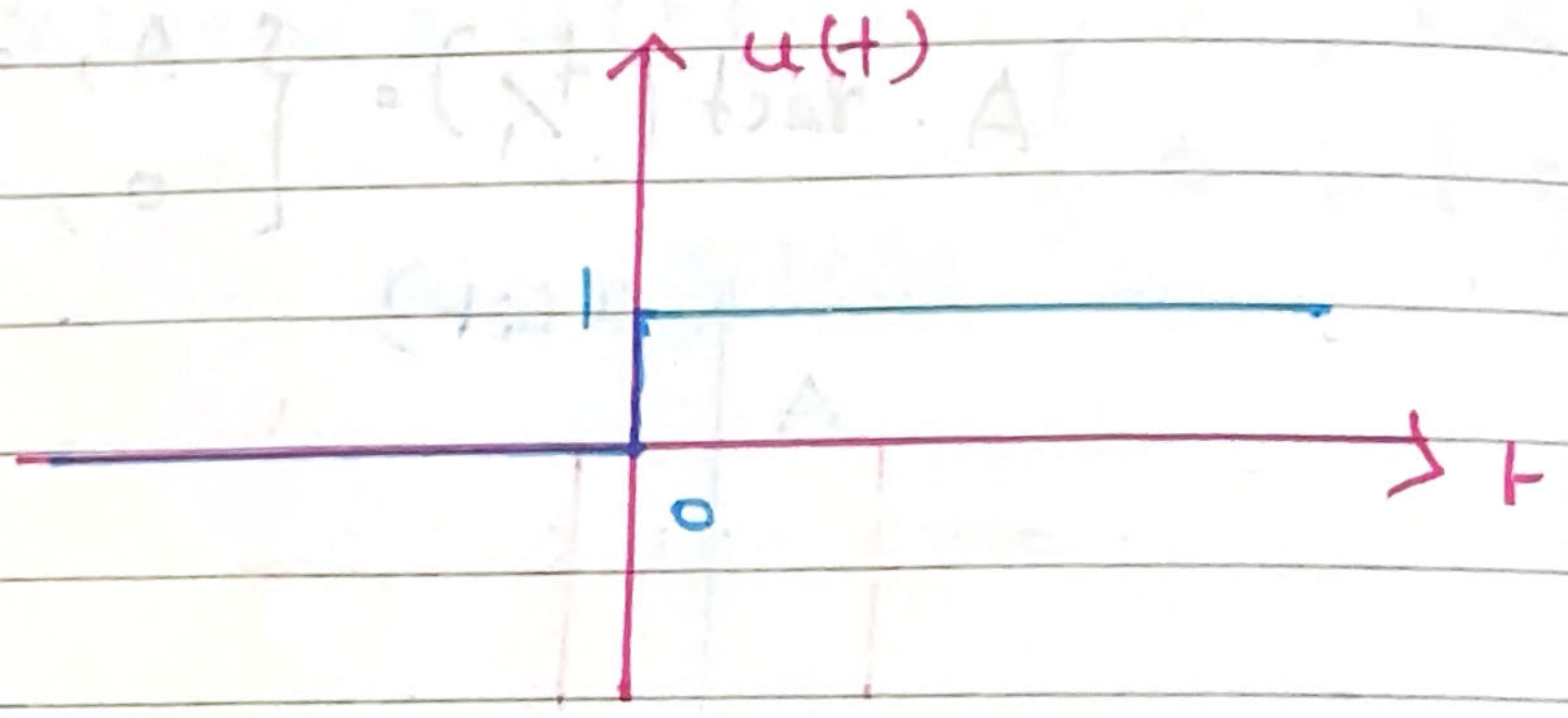
Time reversal signal can be obtained by interchanging left and right hand side positions with respect to Vertical axis (or) Y-axis. If the given

Signal is $x(t)$, then its time reversal form is represented with $\boxed{x(-t)}$ and the operation is called folding (or) mirror Image (or) time reversal.

Eg :

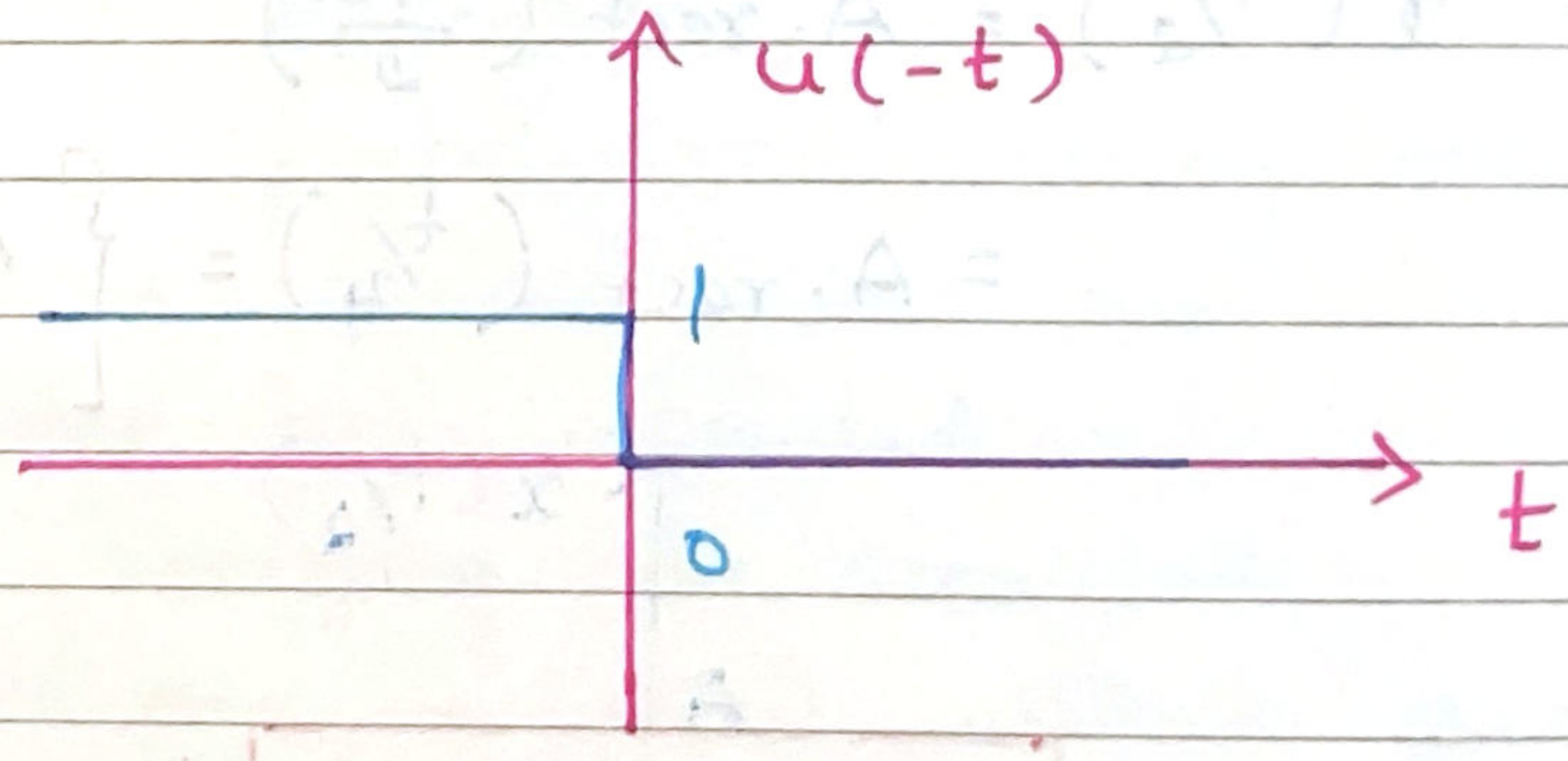
Given: unit step signal $x(t) = u(t)$

$$u(t) = \begin{cases} 1 & ; t \geq 0 \\ 0 & ; t < 0 \end{cases}$$



Time reversal form of $x(t) = x(-t)$

$$x(-t) = u(-t) = \begin{cases} 1 & ; t \leq 0 \\ 0 & ; t \geq 0 \end{cases}$$



Amplitude Shifting operation :-

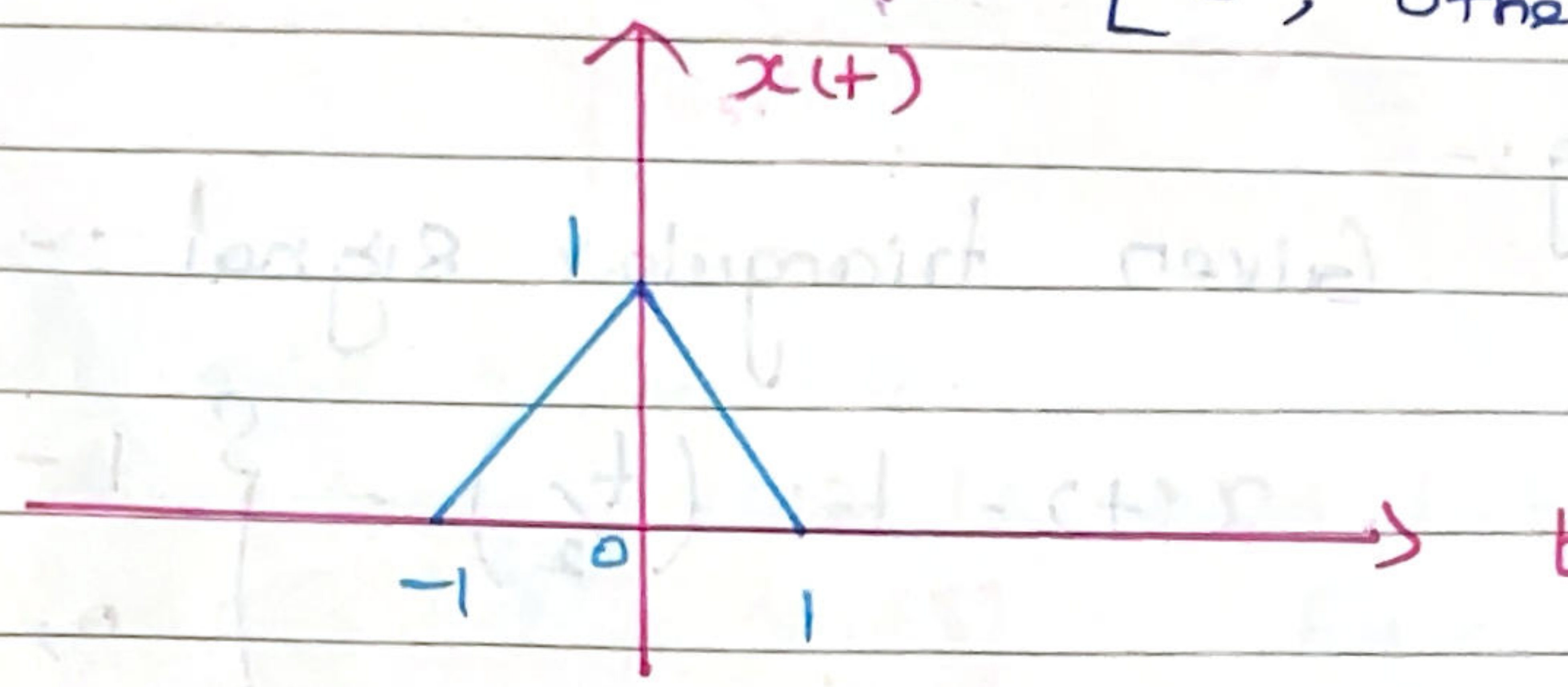
If amplitude shifting operation is applied on a signal $x(t)$, then the signal shifted to up (or) down.

It is represented as $A + x(t)$ where A is constant. It is also known as amplitude (or) Dc level shifter

Eg:-

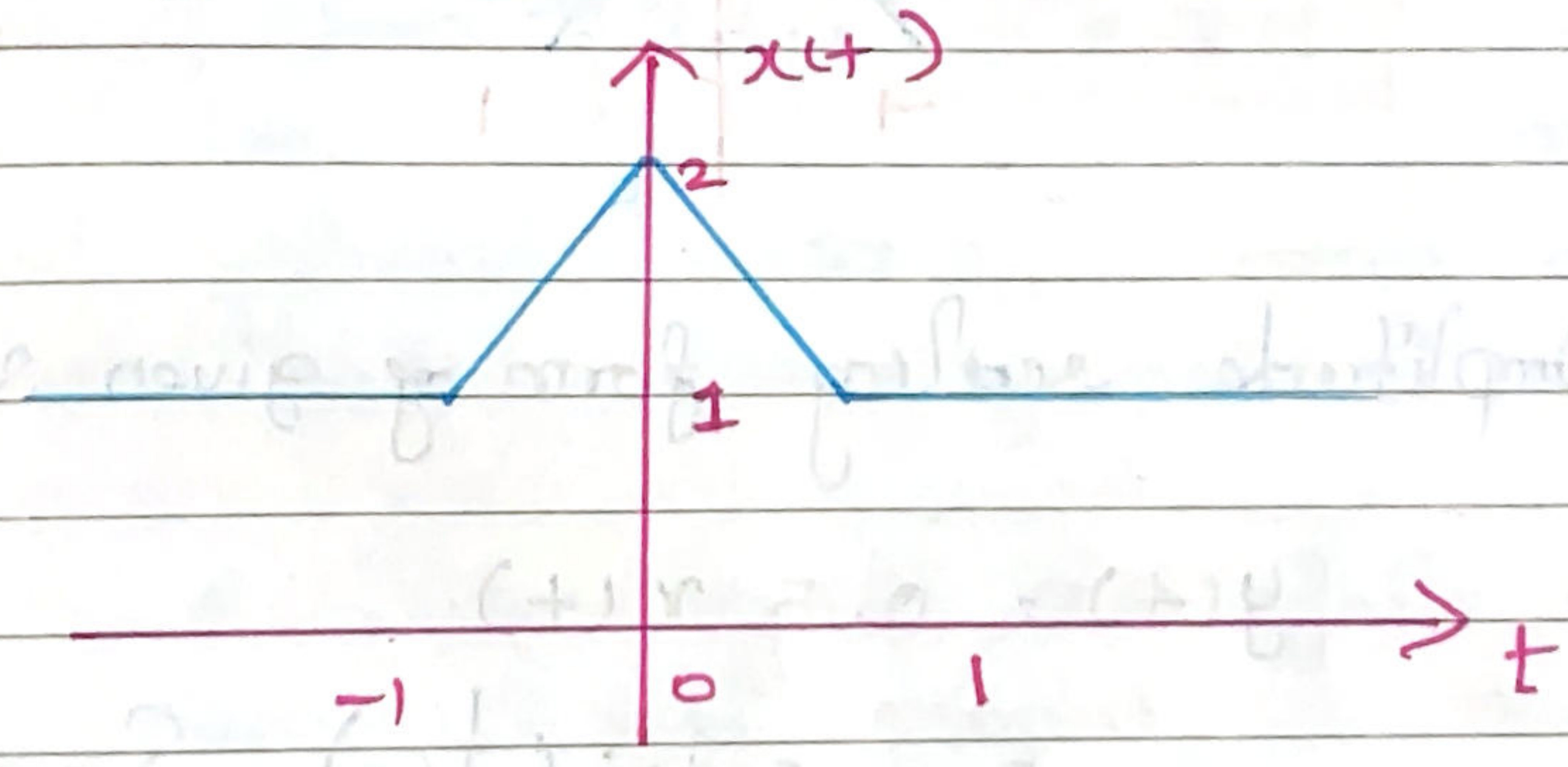
Given Triangular signal.

$$x(t) = tri\left(\frac{t}{2}\right) = \begin{cases} 1-|t| & ; -1 < t < 1 \\ 0, & \text{otherwise.} \end{cases}$$



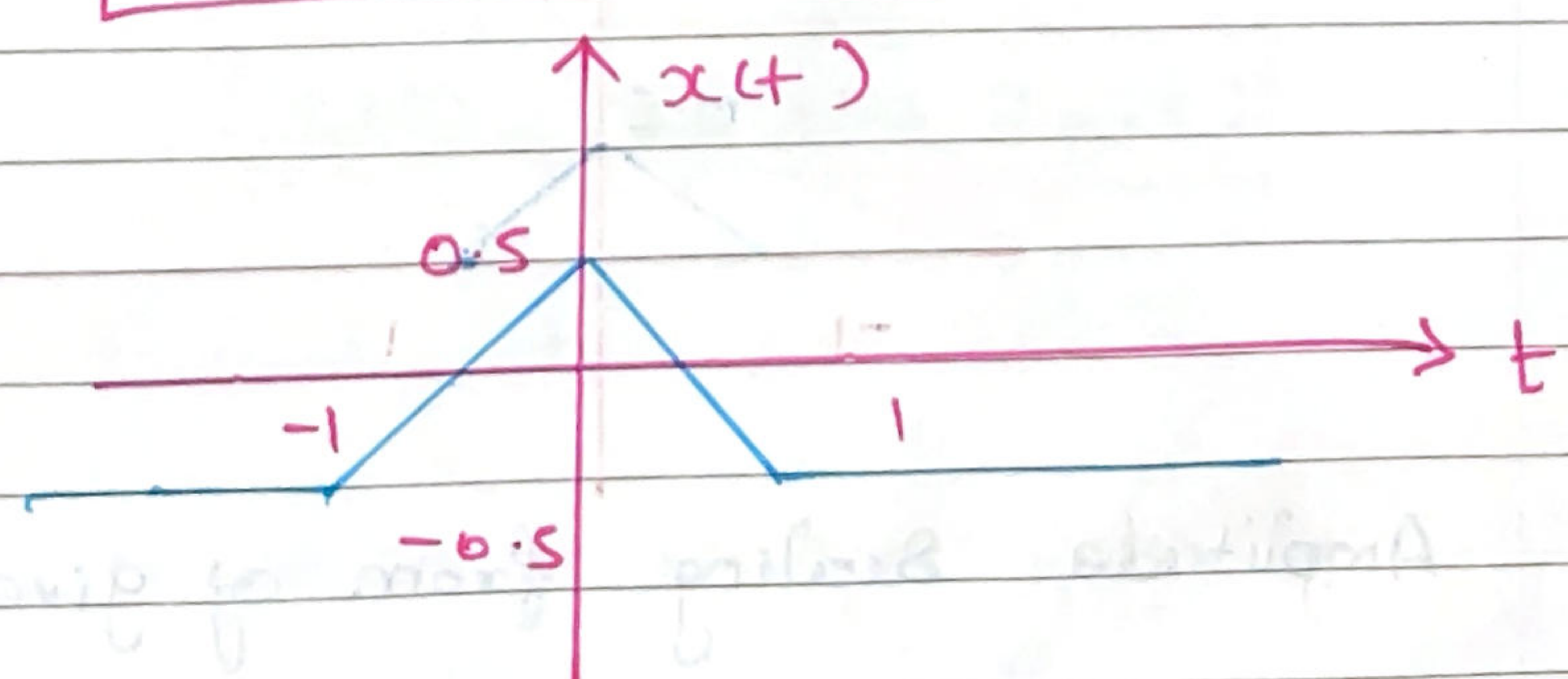
Amplitude shifting of $x(t)$,

$$\boxed{y(t) = 1 + x(t)}$$



Amplitude shifting of $x(t)$,

$$\boxed{y(t) = -0.5 + x(t)}$$



Amplitude scaling operation:-

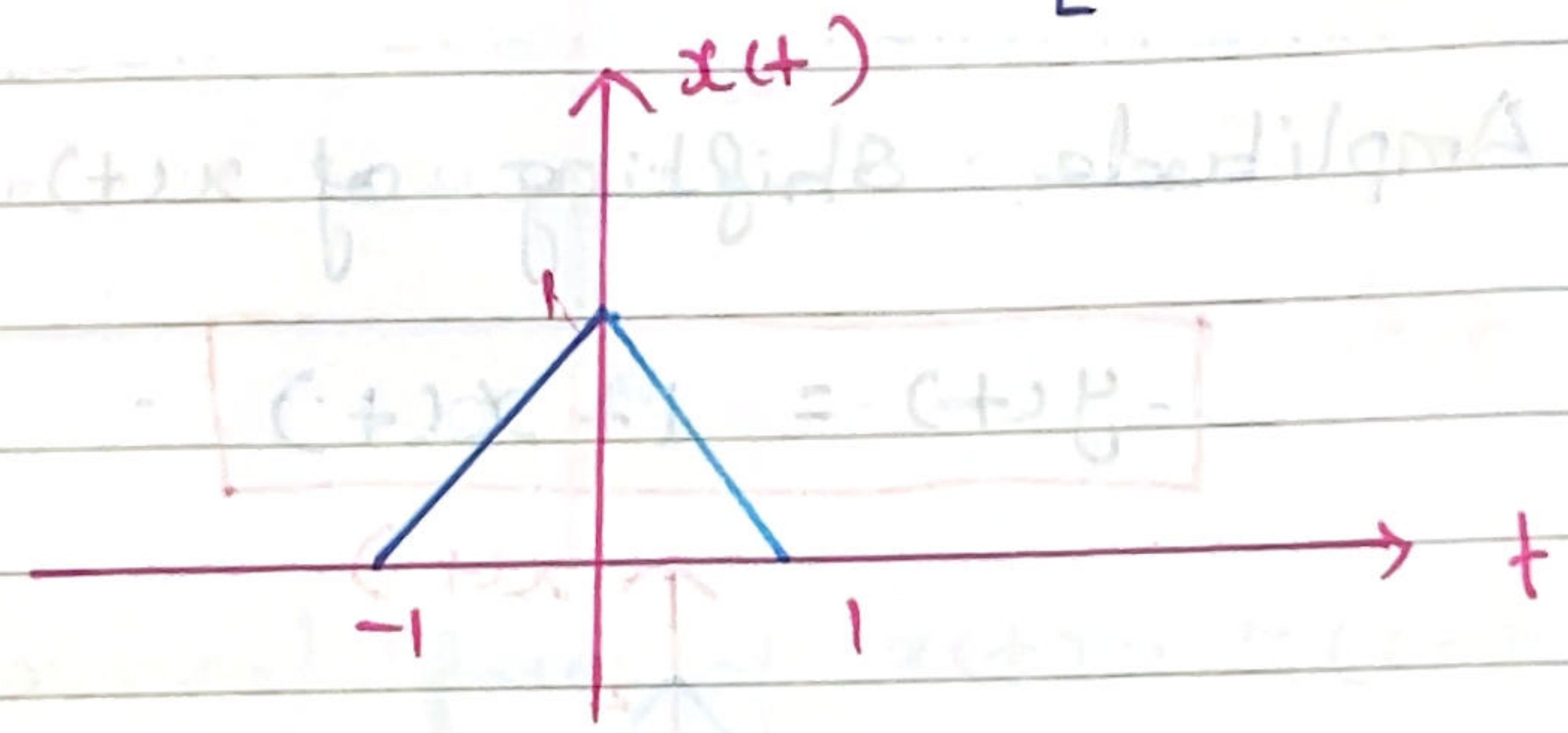
If the amplitude scaling operation is applied on a signal $x(t)$

then the signal amplitude may increase (or) decrease without changing its duration. It is represented with $A x(t)$ where A is amplitude scaling parameter.

Eg:-

Given triangular signal :-

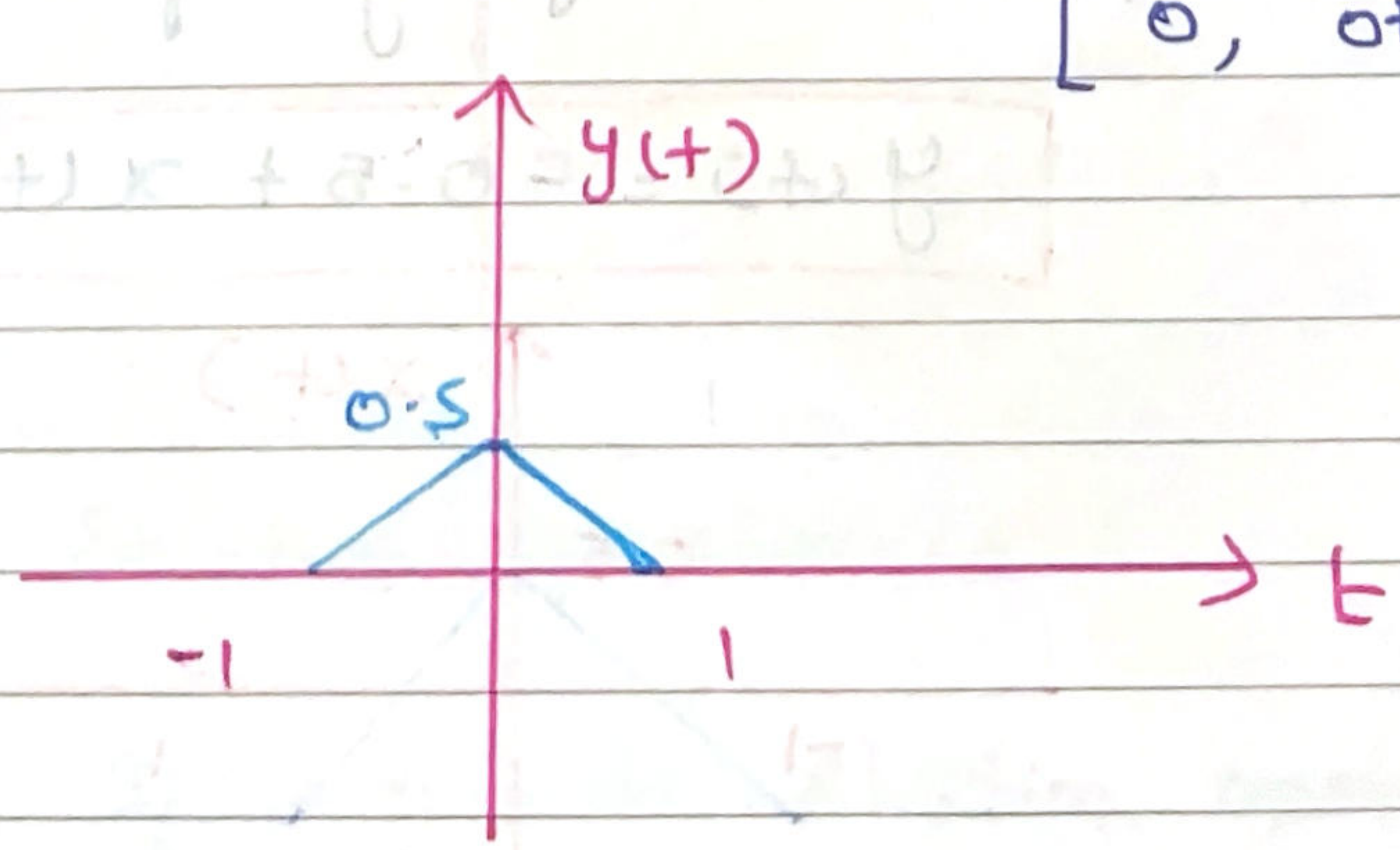
$$x(t) = tri\left(\frac{t}{2}\right) = \begin{cases} 1-|t|, & -1<t<1 \\ 0, & \text{otherwise} \end{cases}$$



Amplitude scaling form of given signal is

$$y(t) = 0.5\, x(t)$$

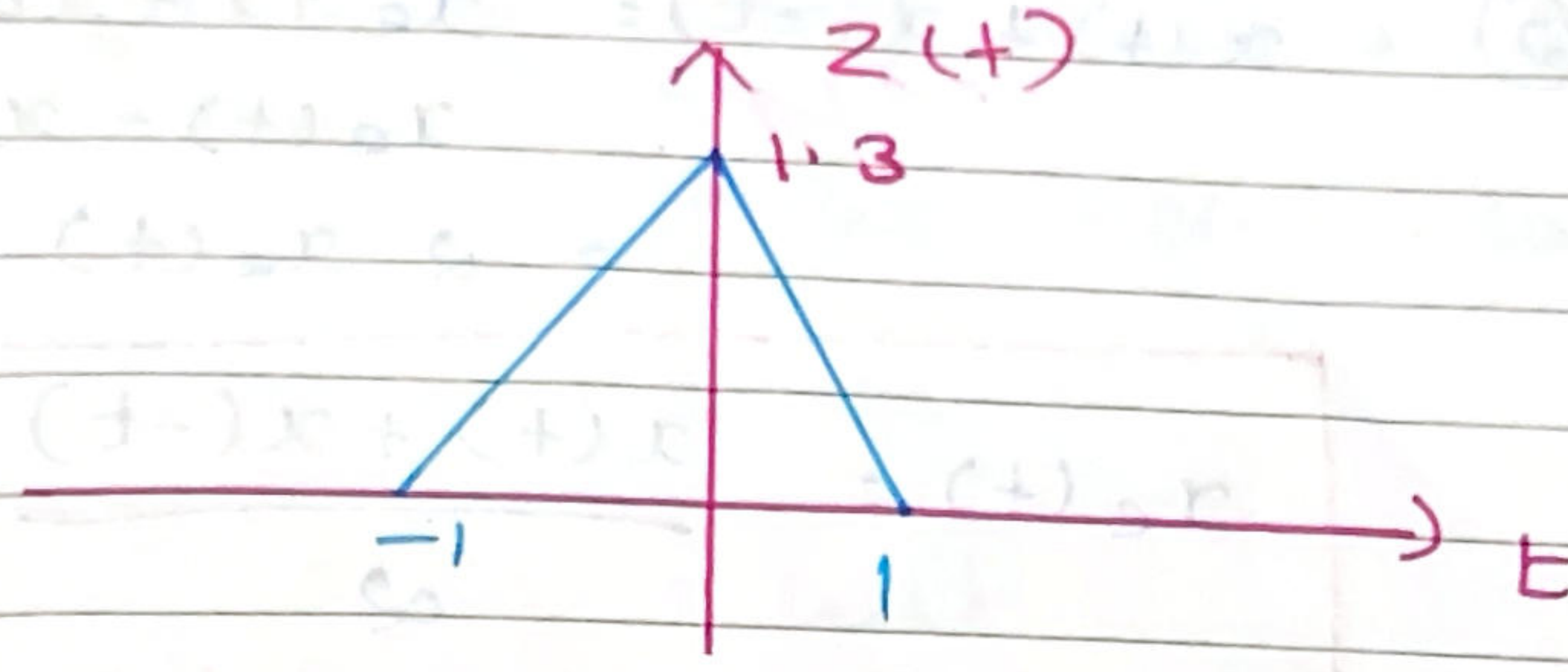
$$= 0.5\, tri\left(\frac{t}{2}\right) = \begin{cases} 0.5(1-|t|); & -1<t<1 \\ 0, & \text{otherwise} \end{cases}$$



Amplitude Scaling from of given signal.

$$z(t) = 1.3\, x(t)$$

$$= 1.3\, tri\left(\frac{t}{2}\right) = \begin{cases} 1.3(1-|t|); & -1<t<1 \\ 0, & \text{otherwise} \end{cases}$$



Even and odd Signal :-

* A signal $x(t)$ is said to be even only when $\boxed{x(-t) = x(t)}$. Even signals are symmetrical about y-axis, hence even signals are called symmetry signals.

* A signal $x(t)$ is said to be odd only when $\boxed{x(-t) = -x(t)}$. Odd signals are anti-symmetrical about y-axis, hence odd signals are called anti-symmetrical signals.

* If the signal $x(t)$ fails to satisfy even and odd property, then the signal is neither odd nor even and it can be expressed as the sum of even signal $x_e(t)$ and odd signal $x_o(t)$

$$\boxed{x(t) = x_e(t) + x_o(t)} \quad \text{— (1)}$$

Replace 't' with '-t'

$$x(-t) = x_e(-t) + x_o(-t)$$

$$\Rightarrow x(-t) = x_e(t) - x_o(t) \quad \text{— (2)}$$

Where $x_e(t)$ = even part of $x(t)$
$x_o(t)$ = odd part of $x(t)$.

① + ② $\quad x(t) + x(-t) = x_e(t) + x_o(t) + x_e(t) - x_o(t)$

$$= 2\, x_e(t)$$

$$\boxed{x_e(t) = \frac{x(t) + x(-t)}{2}}$$

① − ② ⟹ $\quad x(t) - x(-t) = x_e(t) + x_o(t) - (x_e(t) - x_o(t))$

$$= 2\, x_o(t)$$

$$\boxed{x_o(t) = \frac{x(t) - x(-t)}{2}}$$

Eg:-

$$x(t) = \cos t + \sin t + \cos t \sin t$$

1. Find the even and odd components

Given:-

$$x(t) = \cos t + \sin t + \cos t \sin t .$$

W.K.T :- $\quad x_e(t) = \frac{1}{2}(x(t) + x(-t))$

$$x_o(t) = \frac{1}{2}(x(t) - x(-t))$$

$$x(-t) = \cos(-t) + \sin(-t) + \cos(-t)\sin(-t)$$

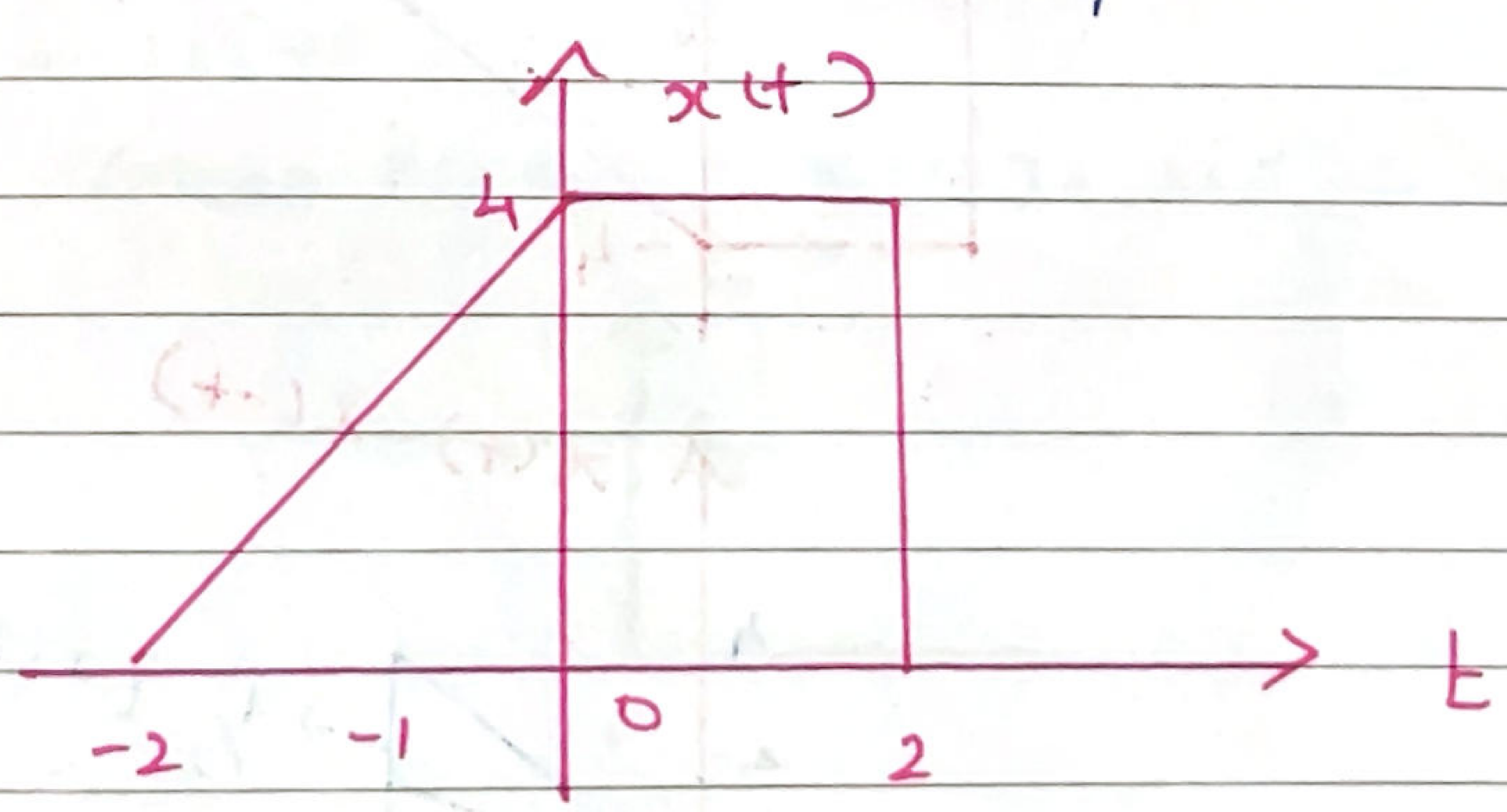
$\because \cos(-\theta) = \cos\theta \; ; \; \sin(-\theta) = -\sin\theta$

$$x(-t) = \cos t - \sin t - \cos t \sin t$$

$$x_e(t) = \frac{1}{2}\{\cos t + \cancel{\sin t} + \cos t \cancel{\sin t}$$
$$+ \cos t - \cancel{\sin t} - \cos t \cancel{\sin t}\}$$
$$= \frac{1}{2}(2\cos t)$$
$$x_e(t) = \cos t$$

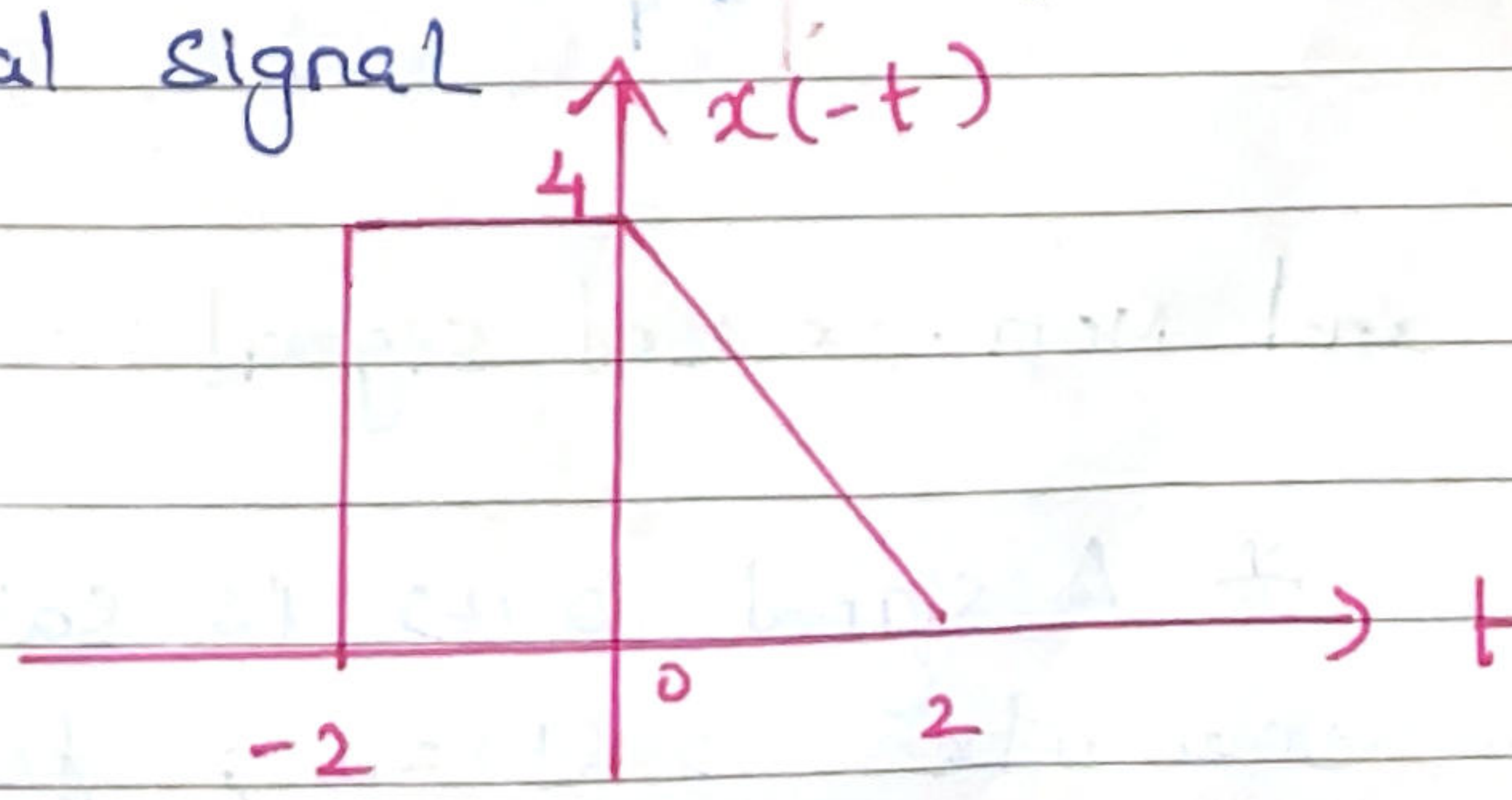
$$x_o(t) = \frac{1}{2}\{\cancel{\cos t} + \sin t + \cos t \sin t$$
$$- \cancel{\cos t} + \sin t + \sin t \cos t\}$$
$$= \frac{1}{2}\{2\sin t + 2\cos t \sin t$$
$$= \frac{2}{2}(\sin t + \cos t \sin t)$$
$$x_o(t) = \sin t + \sin t \cos t$$

2. Find even and odd components:-

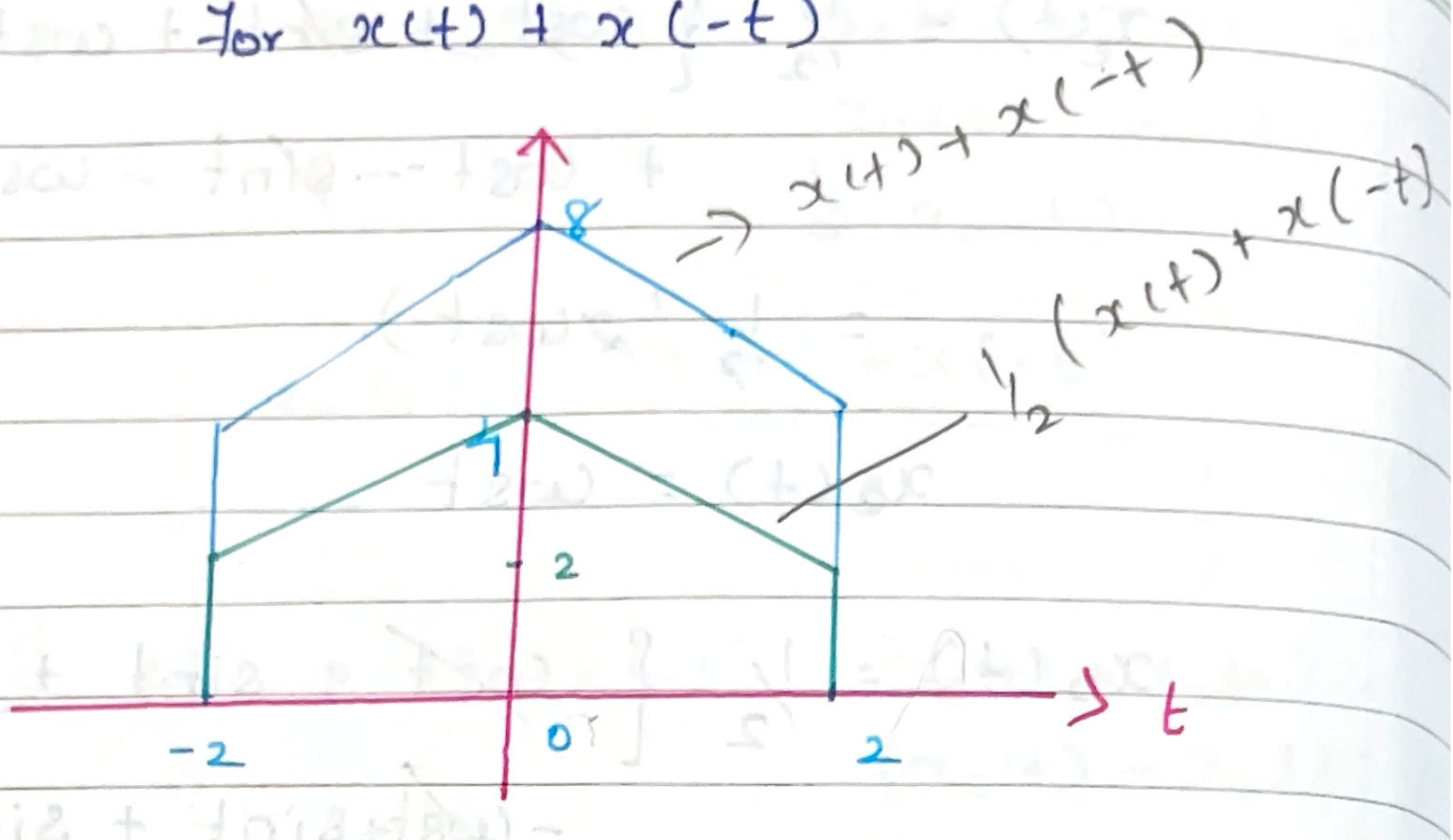


Solution:-

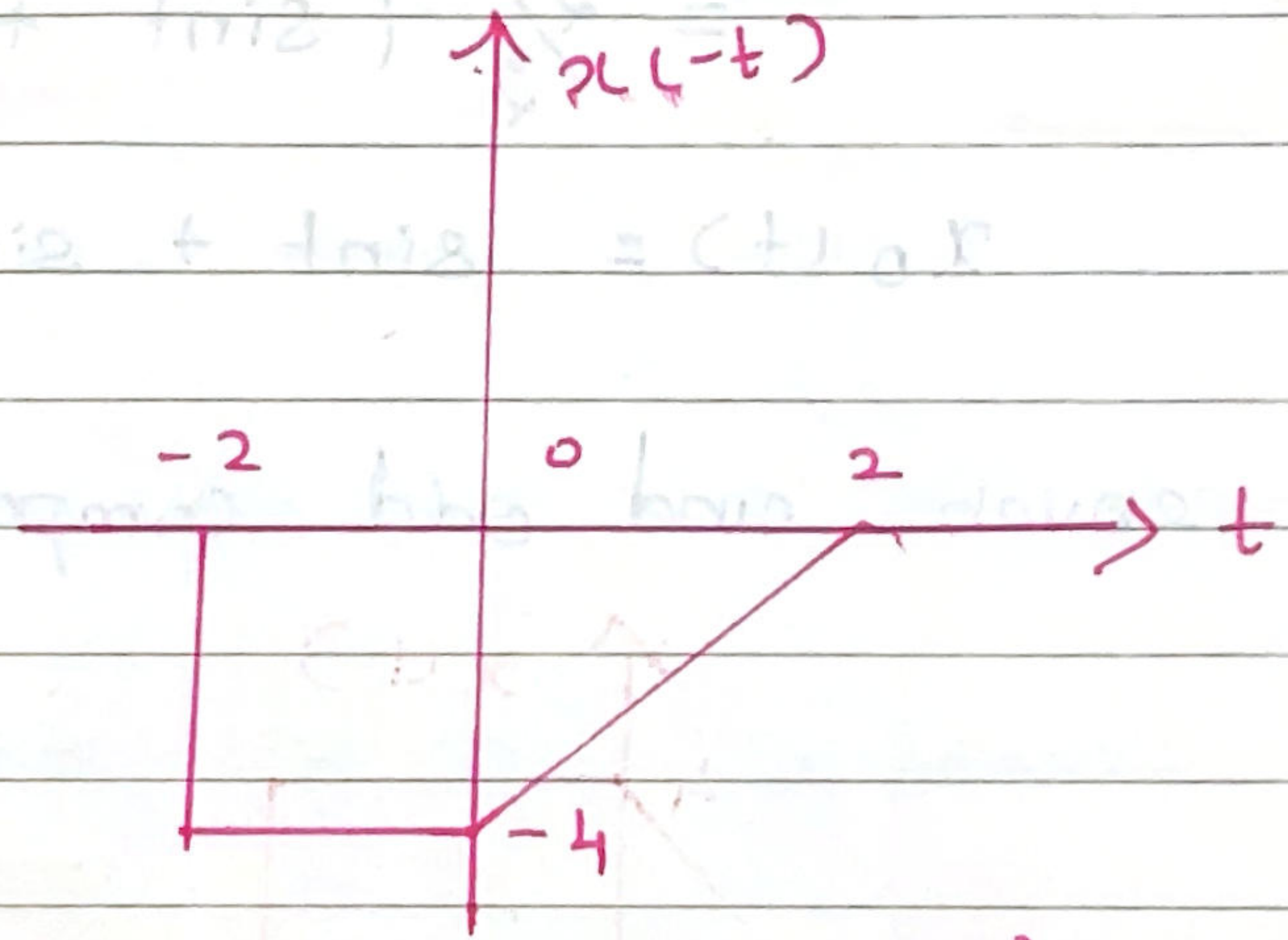
For even, time folded signal (or) reversal signal

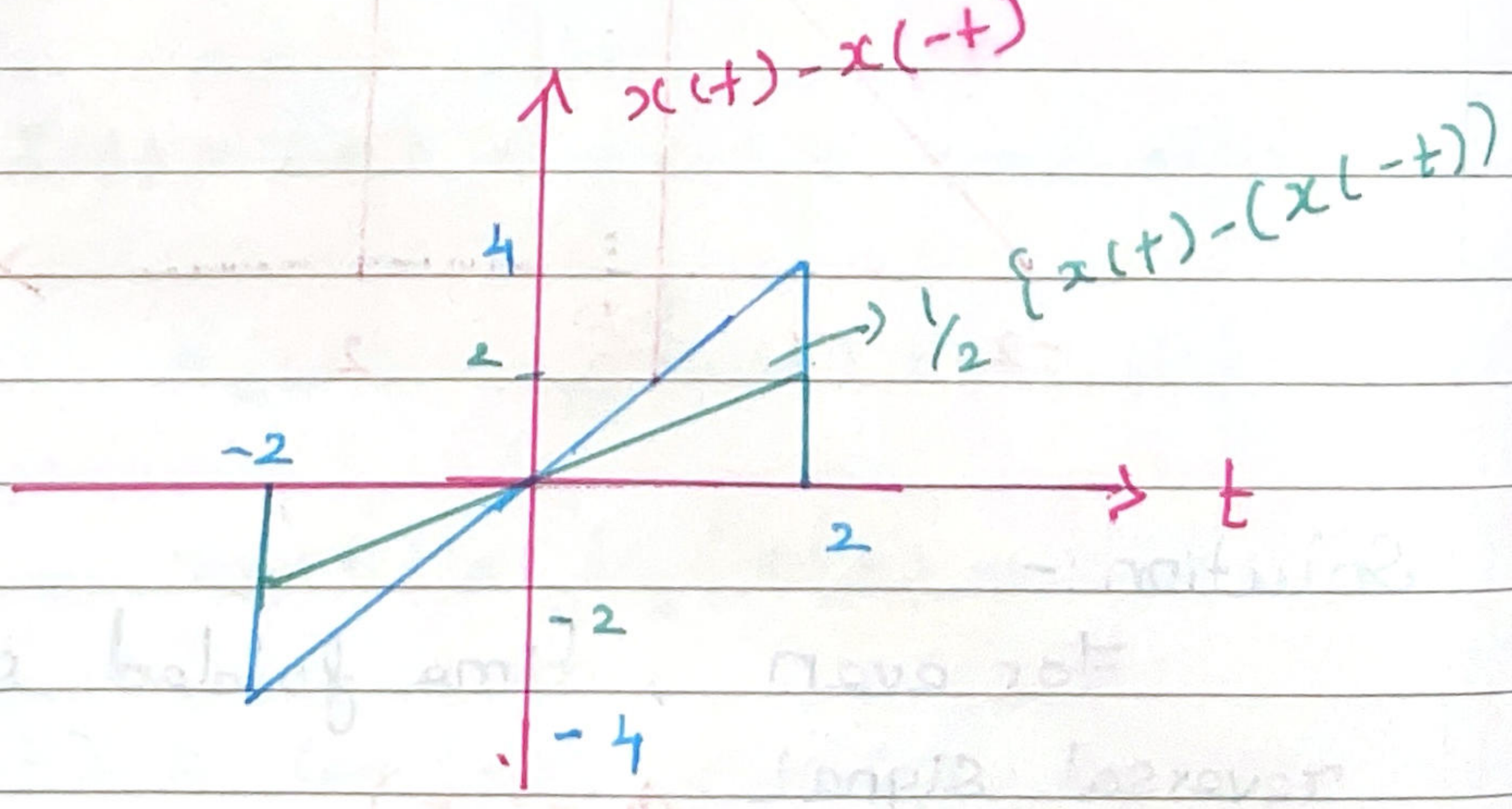


For $x(t) + x(-t)$



For odd → amplitude folding of the time folding signal.





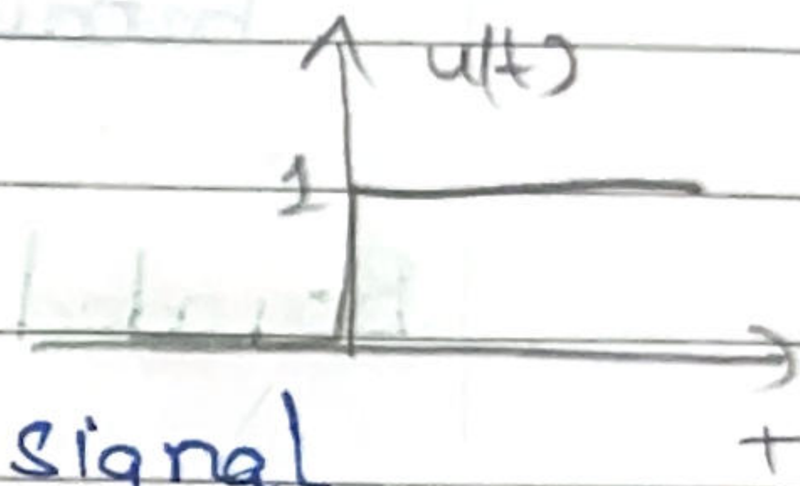
Causal and Non-causal signal :-

* A signal $x(t)$ is said to be casual only when $x(t) = 0$; for $t < 0$, that means casual signals are right sided

* A Signal $x(t)$ is said to be anti casual only when $x(t) = 0$ for $t > 0$, that mean anti causal signal are left sided.

* A signal $x(t)$ is said to be non-causal only when $x(t) \neq 0$ for $t < 0$ ie, non causal signals may be both ~~left~~ sided. all anti causal signals come under non-causal signals.

Eg :-



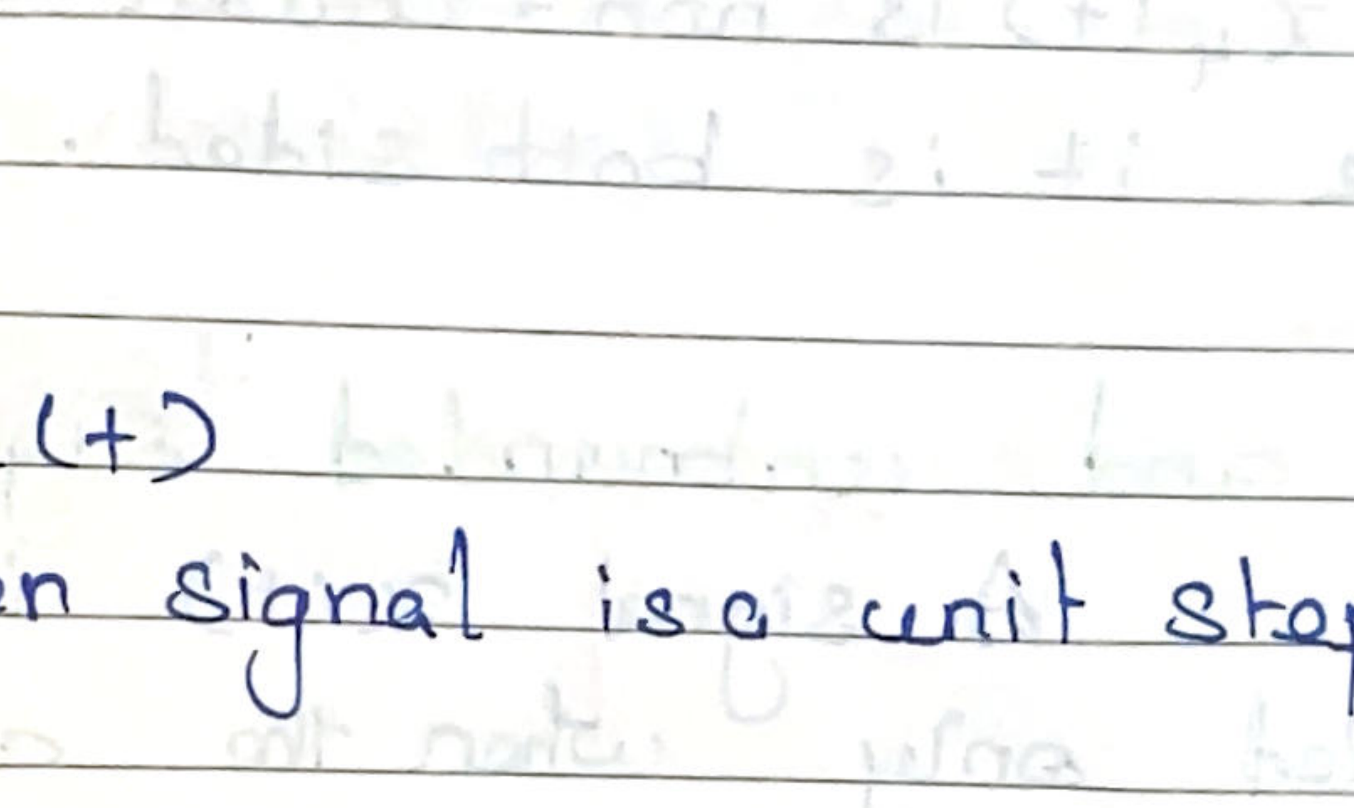
a) $x_1(t) = u(t)$

Given signal is a unit step signal

$\therefore$ $x_1(t)$ = causal signal because it is right sided.

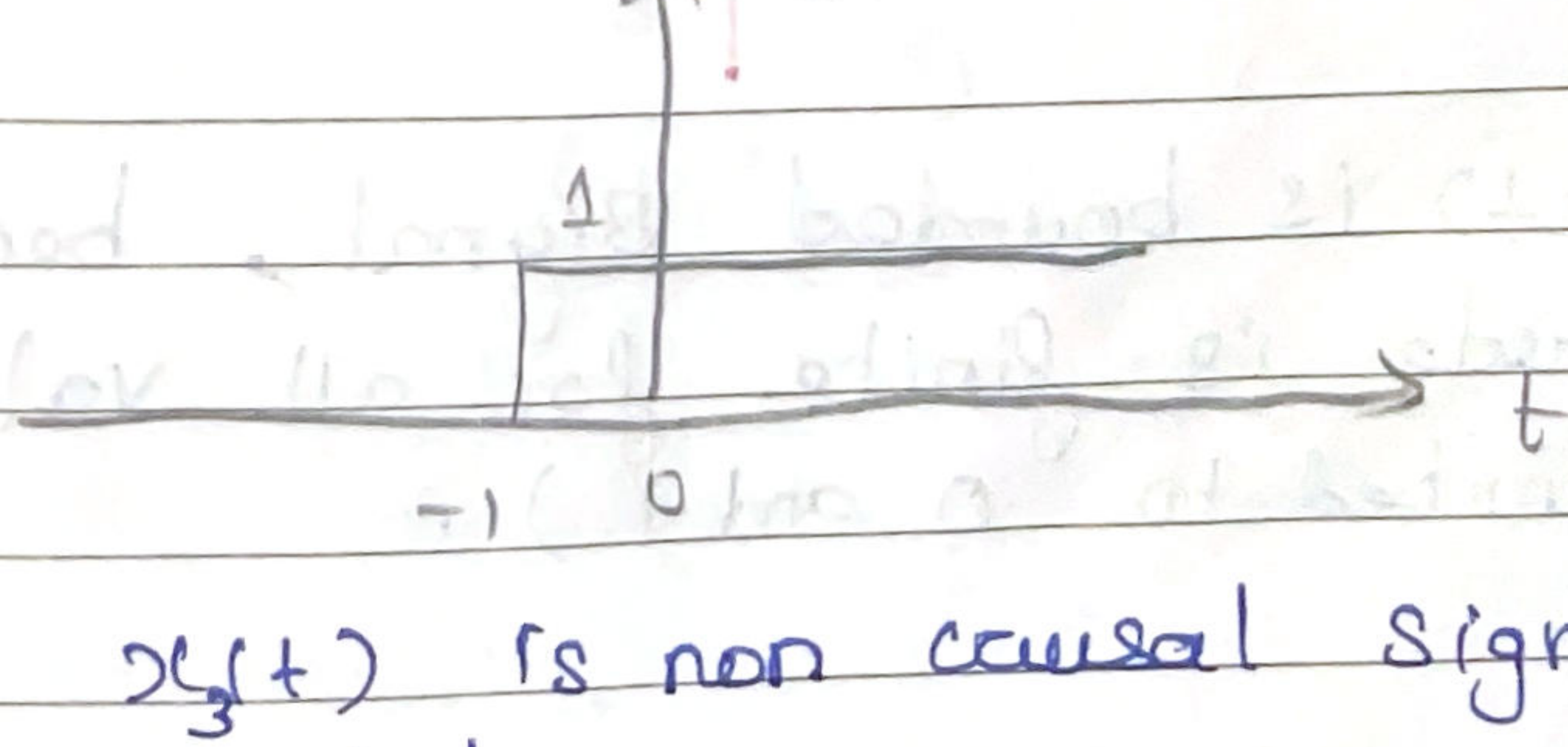
b) $x_2(t) = u(-t)$

Given Signal $x_2(t) = u(-t)$.



$\therefore$ $x_2(t)$ is anti-causal (or) non causal signal because it is left sided.

c) $x_3(t) = u(t+1)$



$x_3(t)$ is non causal signal, because it is both sided.

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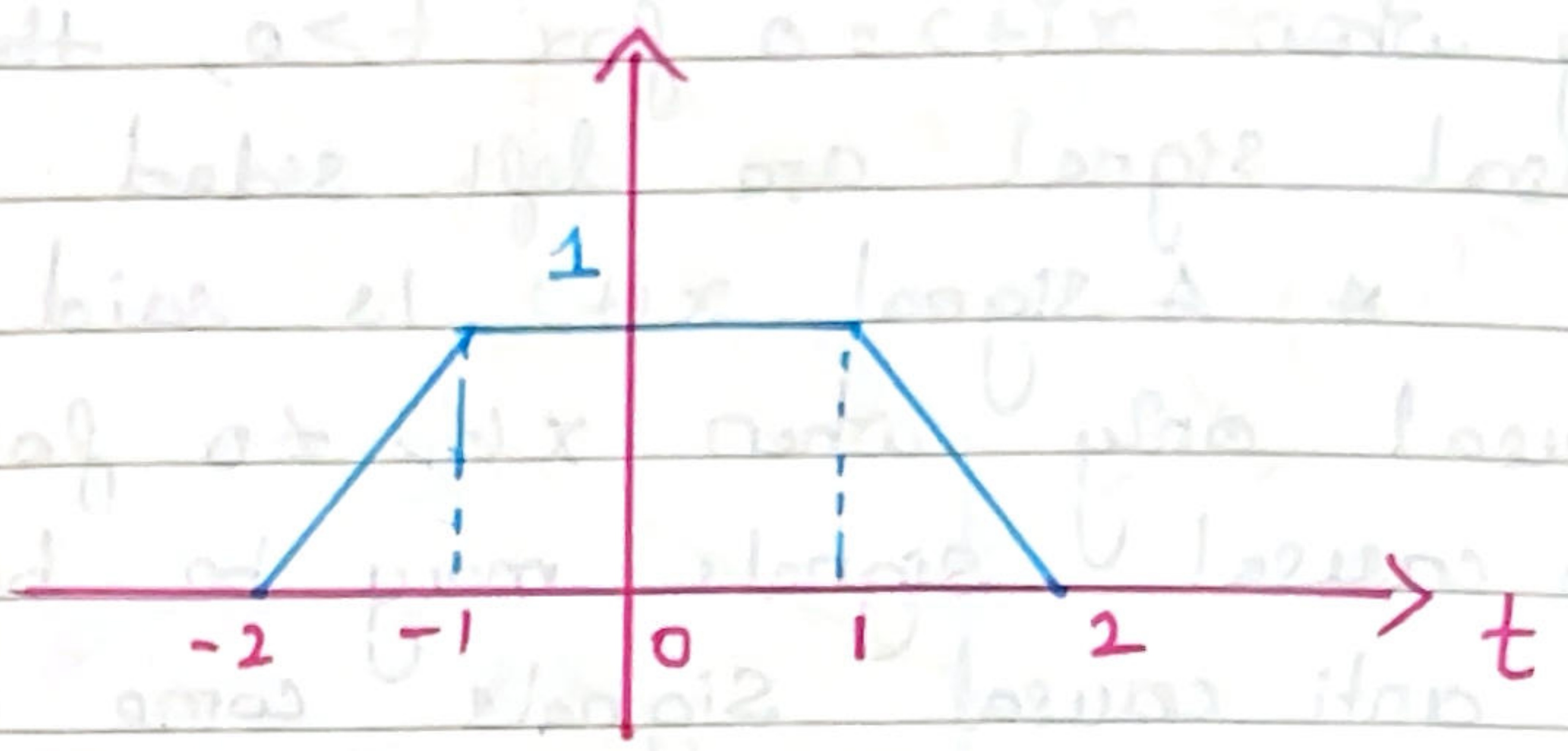
d) $x_4(t) = r(t+2) - r(t+1) - [r(t-1) - r(t-2)]$



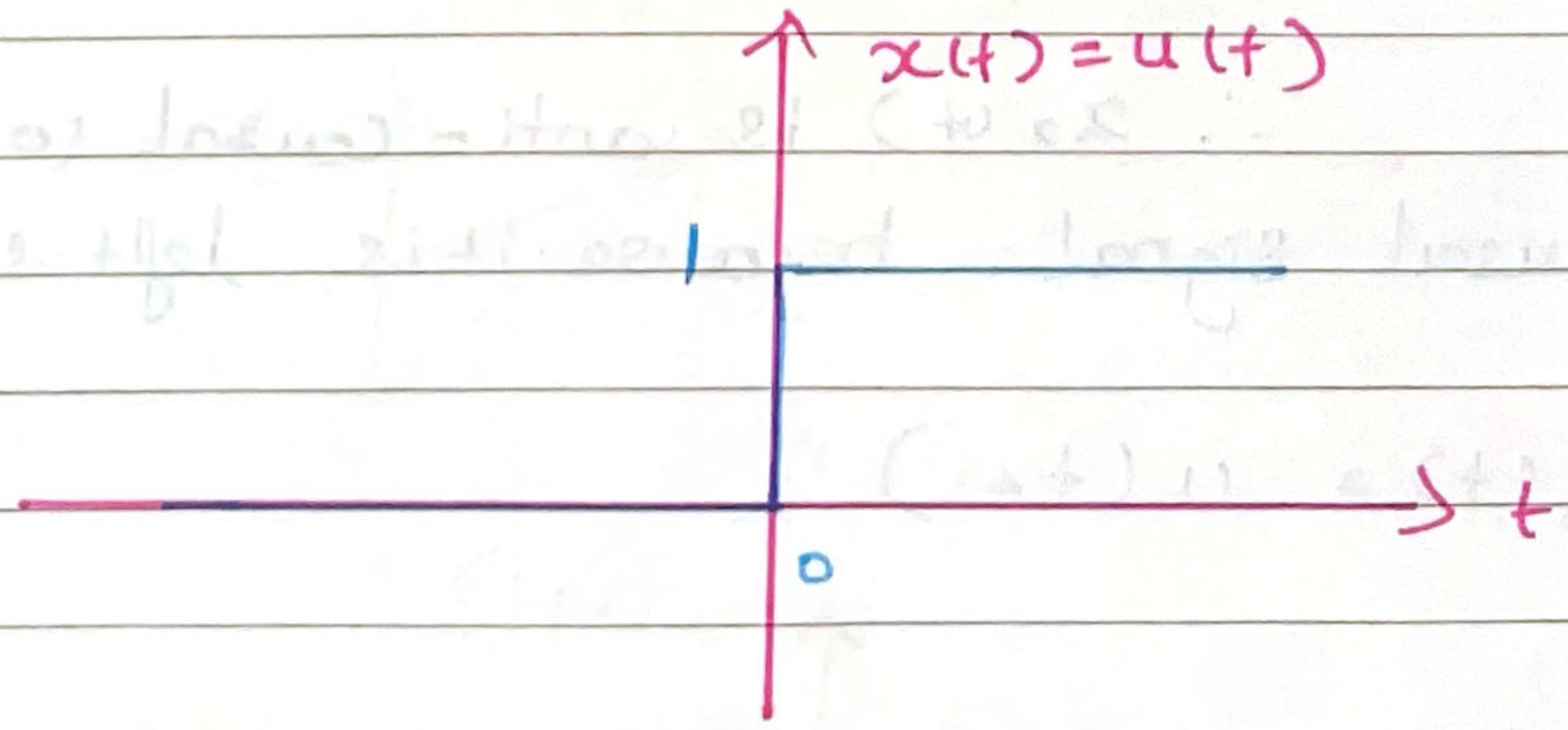
$x_4(t)$ is non-causal signal, because it is both sided.

Bounded and unbounded signals :-

A signal $x(t)$ is said to be bounded only when the amplitude of $x(t)$ is finite for all values of 't' over the range $-\infty \le t \le \infty$. Condition for a bounded signal is $\boxed{|x(t)| < \infty}$, $\boxed{-\infty \le t \le \infty}$

Eg :- a) $x_1(t) = u(t)$

Given : unit step signal,



$x_1(t)$ is bounded signal, because its amplitude is finite for all values of t (limited to 0 and 1).

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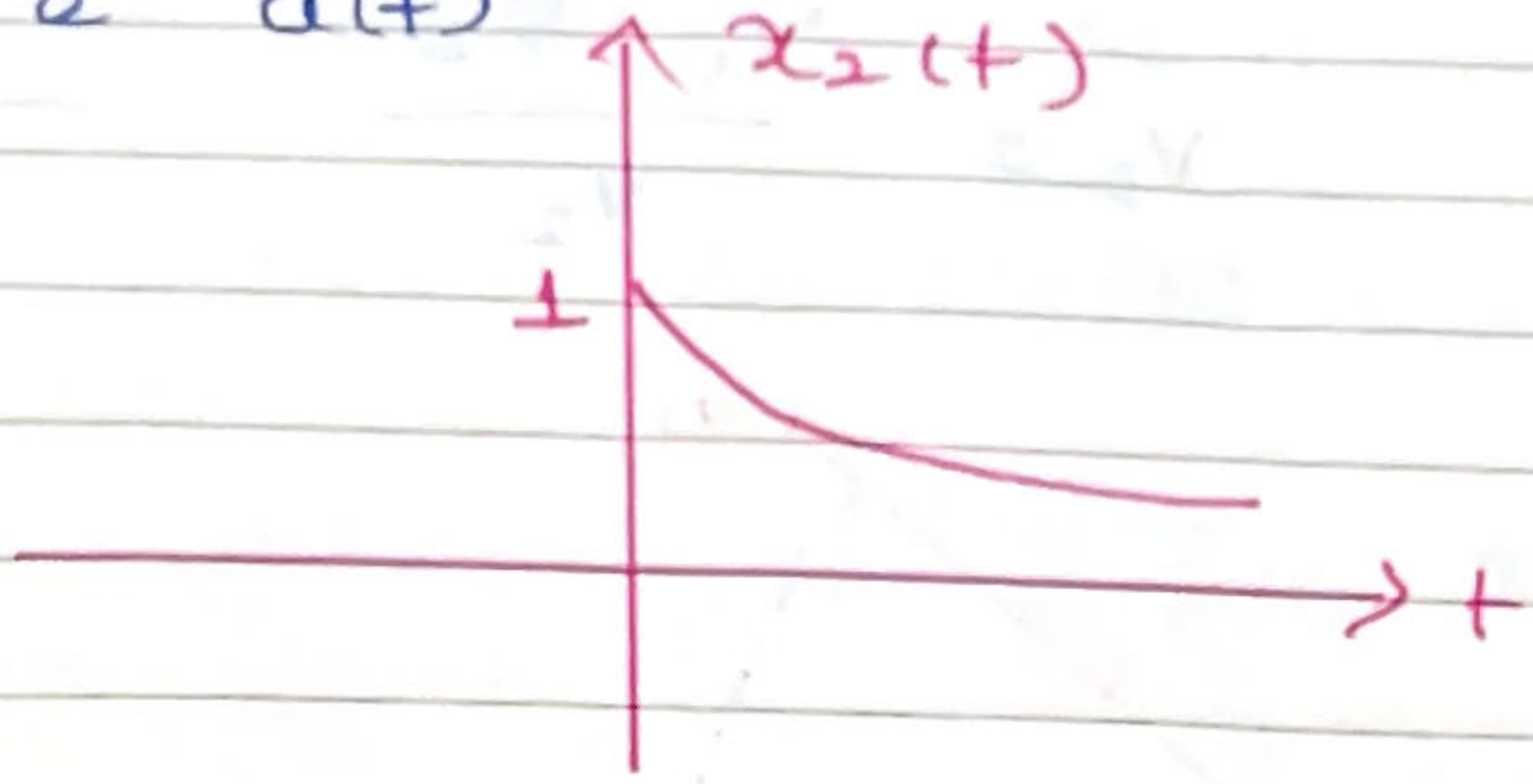
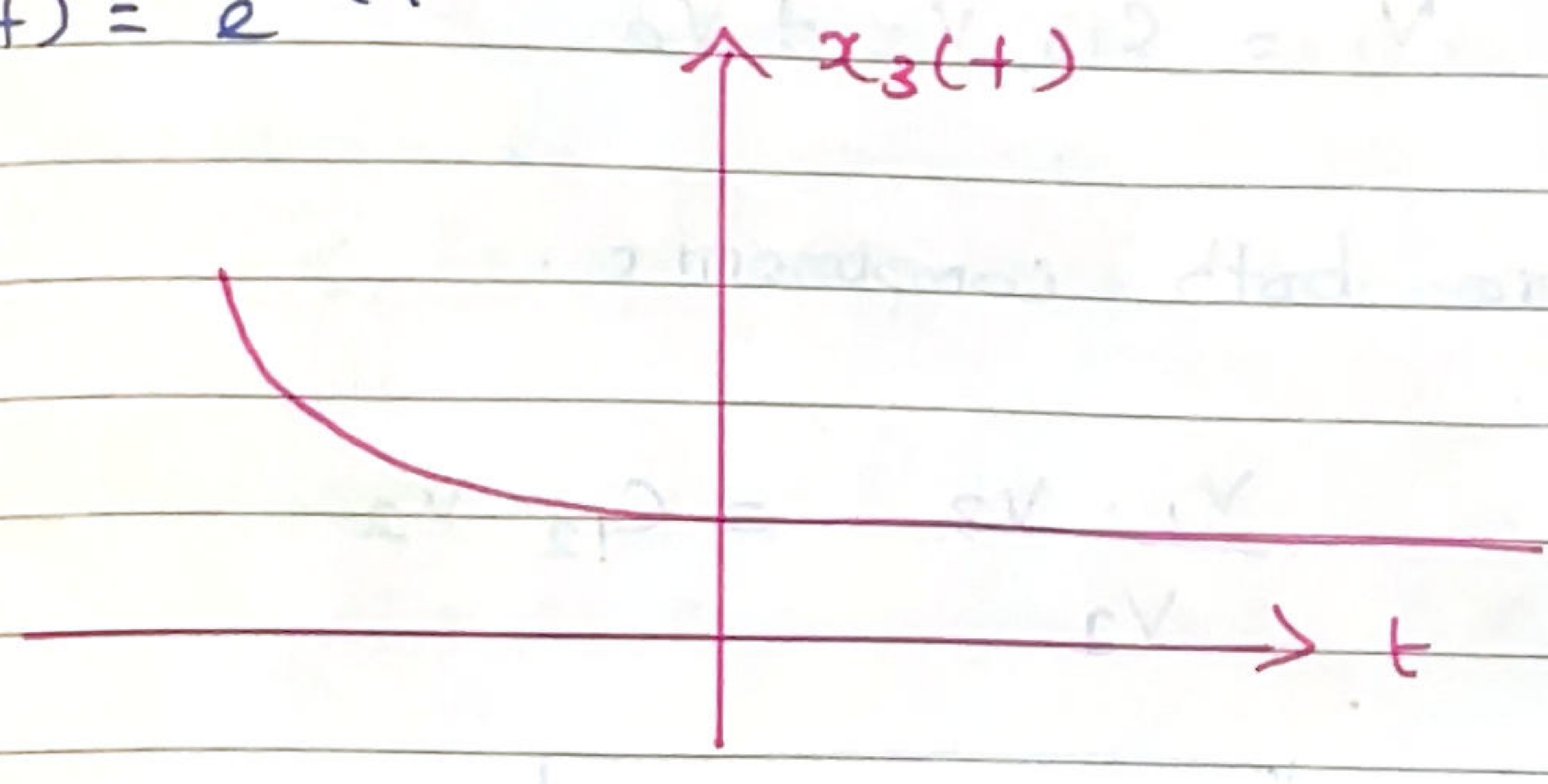
b) $x_2(t) = e^{-2t} u(t)$



$x_2(t)$ is bounded signal, because its amplitude is finite for all values of t (limited to 0 and 1)

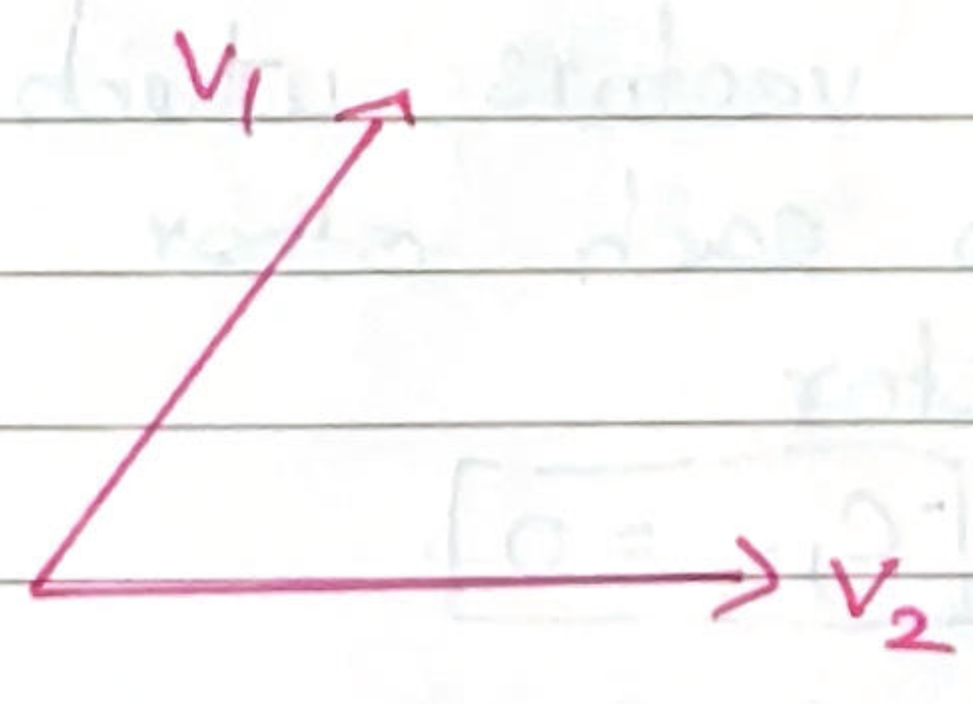
c) $x_3(t) = e^{-2t}$



$x_3(t)$ is unbounded signal, because its amplitude is infinity at $t = -\infty$.

Analogy b/w vectors and signals :-

Vectors can be defined as a quantity, which has both magnitude & direction.



The component of vector v_1 along v_2 is given by

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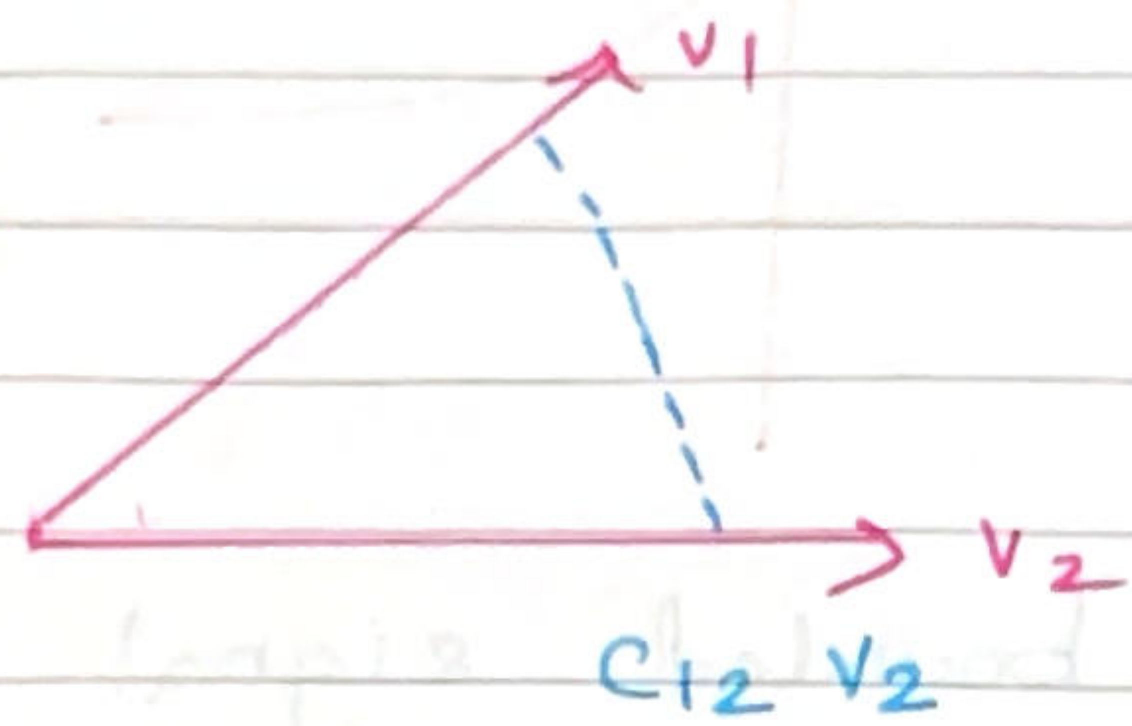
$$V_2 = \frac{V_1 \cdot V_2}{V_2} \quad — (1)$$



$$V_1 \cong C_{12} V_2 \quad — (2)$$

$$V_1 = C_{12} V_2 + V_e$$

Compare both components.

$$\frac{V_1 \cdot V_2}{V_2} = C_{12} \cdot V_2$$

$$\boxed{C_{12} = \frac{V_1 \cdot V_2}{V_2^2}}$$

when two vectors perpendicular to each other (or) dot product is '0'

$$\boxed{C_{12} = 0}$$

Signals :-

* The vectors which are Mutually perpendicular to each other they are called orthogonal vector.

$$\boxed{C_{12} = 0}$$

lets take two Signal $x_1(t)$ and $x_2(t)$. with limits.

$$t_1 < t < t_2$$

$$x_1(t) \cong C_{12}\, x_2(t)$$

$$x_1(t) = C_{12}\, x_2(t) + x_e(t)$$

error signal :-

$$\boxed{x_e(t) = x_1(t) - C_{12}\, x_2(t)}$$

Select C_{12} such that $x_e(t)$ as minimum as ~~as much as~~ possible.

The error canbe Negative (or) positive one.

Generally to minimise the error function we consider a function.

$\boxed{\text{* Mean Square error}}$.

MEAN Square error :-

It is represented as $(\mathcal{E})$

In every approximation process, there will be some error and error due to the approximation can be computed through Mean square error. (MSE). It is denoted a $(\mathcal{E})$ and It can be computed from the formula.

$$\boxed{\mathcal{E} = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} [x_e(t)]^2\, dt}$$

minimisation of MSE :-

w.k.r $x_e(t) = x_1(t) - C_{12}\, x_2(t)$

$$\mathcal{E} = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} [x_1(t) - C_{12} x_2(t)]^2 \, dt$$

$$= \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} [x_1^2(t) + C_{12}^2 x_2^2(t) - 2 C_{12} x_1(t) x_2(t)] \, dt$$

to minimise the mean square error. Compute the constant by differentiating $\mathcal{E}$ with respect to C_{12} is equal to zero

ie, $$\frac{d\mathcal{E}}{dC_{12}} = 0.$$

$$\frac{d\mathcal{E}}{dC_{12}} = \frac{d}{dC_{12}} \left[\frac{1}{t_2 - t_1} \int_{t_1}^{t_2} x_1^2(t) + C_{12}^2 x_2(t) - 2 C_{12} x_2(t) x_1(t) \right] dt$$

Rearranging the terms.

$$= \frac{1}{t_2 - t_1} \left[\int_{t_1}^{t_2} \frac{d}{dC_{12}} x_1^2(t) \, dt + \int_{t_1}^{t_2} \frac{d}{dC_{12}} C_{12}^2 x_2(t) \, dt - \int_{t_1}^{t_2} \frac{d}{dC_{12}} 2 C_{12} x_1(t) x_2(t) \, dt \right.$$

$$= \frac{1}{t_2 - t_1} \left[0 + \int_{t_1}^{t_2} 2 C_{12} x_2^2(t) \, dt - 2 \int_{t_1}^{t_2} x_1(t) x_2(t) \, dt \right]$$

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now $\frac{d\varepsilon}{dC_{12}} = 0$

$$\frac{1}{t_2 - t_1}\left[2\int_{t_1}^{t_2} C_{12}\, x_2^2(t)\,dt - 2\int_{t_1}^{t_2} x_1(t)\, x_2(t)\,dt\right] = 0$$

$$\cancel{2}\int_{t_1}^{t_2} C_{12}\, x_2^2(t)\,dt = \cancel{2}\int_{t_1}^{t_2} x_1(t)\, x_2(t)\,dt$$

$$C_{12}\int_{t_1}^{t_2} x_2^2(t)\,dt = \int_{t_1}^{t_2} x_1(t)\, x_2(t)\,dt$$

$$\boxed{C_{12} = \frac{\int_{t_1}^{t_2} x_1(t)\, x_2(t)\,dt}{\int_{t_1}^{t_2} x_2^2(t)\,dt}}$$

Condition for orthogonal signals :-

If the signals are orthogonal then the scalar (or) dot product is '0'

ie) $C_{12} = 0$

$$\Rightarrow \boxed{\int_{t_1}^{t_2} x_1(t)\, x_2(t)\,dt = 0}$$

We know that signals are analogous to vectors (ie) if two signal $x_1(t)$ & $x_2(t)$ are orthogonal over the intervals (t_1, t_2) then

$$\boxed{\int_{t_1}^{t_2} x_1(t)\, x_2(t)\, dt = 0}$$

It is the condition for orthogonal signal.

NOTE:-

* All sinusoidal signals, $\sin(\omega t)$ and $\cos(\omega t)$ are orthogonal signals.

* All complex exponential signals, $x(t) = Ae^{-j\omega t}$ & $y(t) = Ae^{j\omega t}$ are orthogonal signals.

Eg:-

(1) Show that the following signals are orthogonal over an interval $[0, 1)$, $x_1(t) = 2$, $x_2(t) = \sqrt{3}\,(1-2t)$.

Solution:-

W.K.T $\int_{t_1}^{t_2} x_1(t)\, x_2(t)\, dt = 0$

$$\int_0^1 \{2 \cdot \sqrt{3}\,(1-2t)\}\, dt = 0$$

$$\int_0^1 (2\sqrt{3} - 4\sqrt{3}\,t)\, dt = 0$$

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$$= \int_0^1 2\sqrt{3}\, dt - \int_0^1 4\sqrt{3}\, t\, dt$$

$\int x \cdot dx = x^2/2$

$$= \left[2\sqrt{3}\, t\right]_0^1 - \left[4\sqrt{3}\, \frac{t^2}{2}\right]_0^1$$

$$= 2\sqrt{3}\,[1-0] - 4\sqrt{3}\,[1/2 - 0]$$

$$= 2\sqrt{3} - 2\sqrt{3}$$

$$= 0$$

Hence the given functions are orthogonal over a interval $[0, 1]$

Orthogonal Signal space :-

Mutually orthogonal function : also called as Mutually orthogonal set :-

Consider a set of 'n' function

$\{x_1(t), x_2(t) \ldots x_n(t)\}$ which are orthogonal to one another over (t_1, t_2)

The condition for orthogonality in complex function

$$\boxed{\begin{aligned} &\int_{t_1}^{t_2} x_1^*(t)\, x_2(t)\, dt = 0 \quad (\text{or}) \\ &\int_{t_1}^{t_2} x_1(t)\, x_2^*(t)\, dt = 0 \end{aligned}}$$

$$\text{if } \{x_1(t), x_2(t) \ldots\ldots x_n(t)\}$$

closed, complex set of orthogonal function

$$\int_{t_1}^{t_2} x_m^*(t)\, x_n(t)\, dt = 0 \quad (\text{or})$$

$$\int_{t_1}^{t_2} x_m(t)\, x_n^*(t)\, dt = 0$$

$m \neq n$

If $m = n$

$$\int_{t_1}^{t_2} x_m^*(t)\, x_m(t)$$

$$\boxed{E_m = \int_{t_1}^{t_2} |x_m(t)|^2} \quad - \text{ Energy formula}$$

$\downarrow$

$x_m(t)$

Problems :-

(1) If $x(t)$ and $y(t)$ are orthogonal, then show that $E_{x+y} = E_x + E_y$ is energy of the signal $x(t) + y(t)$. E_x is the energy of signal $x(t)$ & E_y is the energy of the signal $y(t)$

Solution :-

Given :-

$$E_x = \int_{-\infty}^{\infty} x^2(t)\, dt \ \&$$

$$E_y = \int_{-\infty}^{\infty} y^2(t)dt$$

The energy signal $x_o(t) + y(t)$ is

$$E_{x+y} = \int_{-\infty}^{\infty} (x(t) + y(t))^2 dt$$

$$= \int_{-\infty}^{\infty} x^2(t)dt + \int_{-\infty}^{\infty} y^2(t)dt +$$

$$\int_{-\infty}^{\infty} 2(x(t)\, y(t))dt$$

since $x(t)$ and $y(t)$ are orthogonal

w.k.r $\int_{-\infty}^{\infty} x(t)y(t) = 0$

$$\therefore E_{x+y} = \int_{-\infty}^{\infty} x^2(t)dt + \int_{-\infty}^{\infty} y^2(t)dt$$

$$E_{x+y} = E_x + E_y$$

(2) Prove that the set of exponentials $\{1, e^{\pm j\omega_0 t}, e^{\pm j2\omega_0 t}, \dots\}$ is orthogonal over any interval To

Solution :-

Given:-

The set of exponentials are complex Signals.

W.K.T $\int_{t_1}^{t_2} x_m(t)\, x_n^*(t)\, dt$.

Let $x_m(t) = e^{jm\Omega_0 t}$ &

$x_n(t) = e^{jn\Omega_0 t}$

for any interval T_0

$$\int_{t}^{t+T_0} e^{jm\Omega_0 t} \left(e^{jn\Omega_0 t}\right)^* dt$$

$$= \int_{t}^{t+T_0} e^{j(m-n)\Omega_0 t} dt$$

$$= \frac{1}{j(m-n)\Omega_0} \left[e^{j(m-n)\Omega_0 t}\right]_{t}^{t+T_0}$$

$$= \frac{1}{j(m-n)\Omega_0} \left[e^{j(m-n)\Omega_0 (t+T_0)} - e^{j(m-n)\Omega_0 t}\right]$$

$$= \frac{1}{j(m-n)\Omega_0} e^{j(m-n)\Omega_0 t} \left[e^{j(m-n)\Omega_0 T_0} - 1\right]$$

Since $\Omega_0 T_0 = 2\pi$

$e^{j(m-n)2\pi} = 1$ for any m & n

Hence :

$$\int_{t}^{t+T_0} e^{jm\Omega_0 t} \left(e^{jn\Omega_0 t}\right)^* dt = 0$$

if $m = n$, then

$$\int_{t}^{t+T_0} e^{jm\Omega_0 t} \left(e^{jn\Omega_0 t}\right)^* dt =$$

$$\int_{t}^{t+T_0} dt = T_0$$

$\therefore$

$$\int_{t}^{t+T_0} e^{jm\Omega_0 t} \left(e^{jn\Omega_0 t}\right)^* dt = 0 \text{ for } m \neq n$$

$$= T_0 \text{ for } m = n.$$

$\therefore$ The set of exponentials is orthogonal.

Parseval's relation:-

The total energy (or) power of a signal in the time domain is equal to its total energy (or) power in the frequency domain.

For

Continuous - time Fourier Transform.

For continuous - time a periodic signal $x(t)$ with a fourier transform $X(\omega)$, the energy conservation is represented as.

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega$$

If ~~two~~ dealing with two different signals $x_1(t)$ and $x_2(t)$ the parseval's relation is

$$\int_{-\infty}^{\infty} x_1(t)\, x_2^*(t)\, dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} X_1(j\omega)\, X_2^*(j\omega)\, d\omega$$

Discrete time signals :-

In Discrete time furier transforms where $X(\omega)$ is a Continuous function of frequency with a period of 2π, the relation is:

$$\sum_{n=-\infty}^{\infty} |x(n)|^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} |X(j\omega)|^2 d\omega$$

For finite length N (using DFT) it is translates to

$$\sum_{n=0}^{N-1} |x(n)|^2 = \frac{1}{N} \sum_{N=0}^{N-1} |X(k)|^2$$

Parseval's relation

If $F[x(t)] = X(j\omega)$

then

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega$$

Proof :-

$$|x(t)|^2 = x(t)\, x^*(t)$$

Integrating both sides over $-\infty$ to ∞

w.k.t Energy

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} x(t) \cdot x^*(t)\, dt \quad \text{--- (1)}$$

$$x(t) = F^{-1}[X(j\omega)] = \int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, dt.$$

$$x^*(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} x^*(j\omega)\, e^{-j\omega t}\, dt$$

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} x(t) \left[\frac{1}{2\pi} \int_{-\infty}^{\infty} x^*(j\omega)\, e^{-j\omega t}\, d\omega \right] dt.$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} X^*(j\omega) \left[\int_{-\infty}^{\infty} x(t)\, e^{-j\omega t} \right] d\omega$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} X^*(j\omega)\, X(j\omega)\, d\omega$$

$$\therefore \int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega.$$

Hence proved.